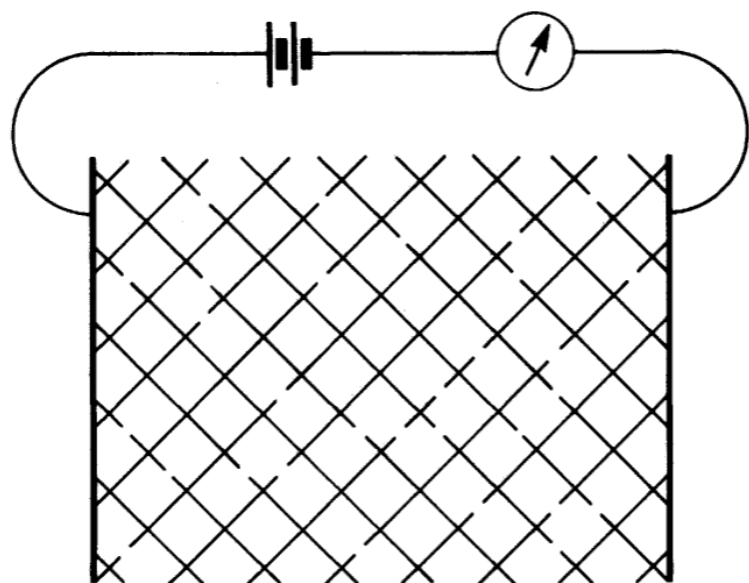


Non-metal to Metal Transitions

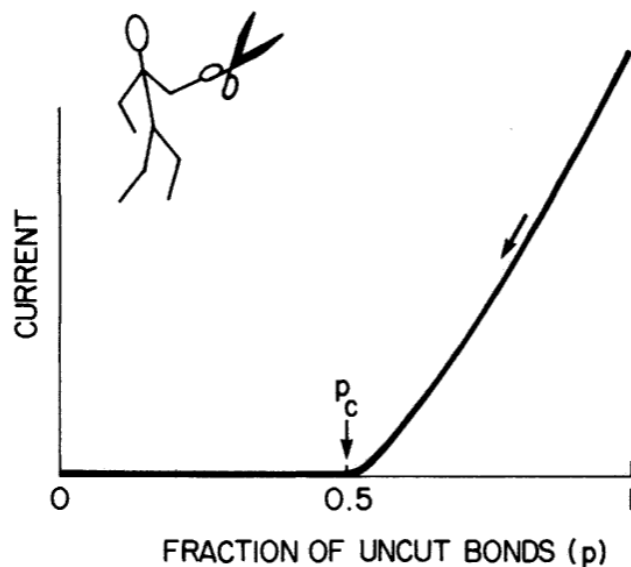
the saga continues ...



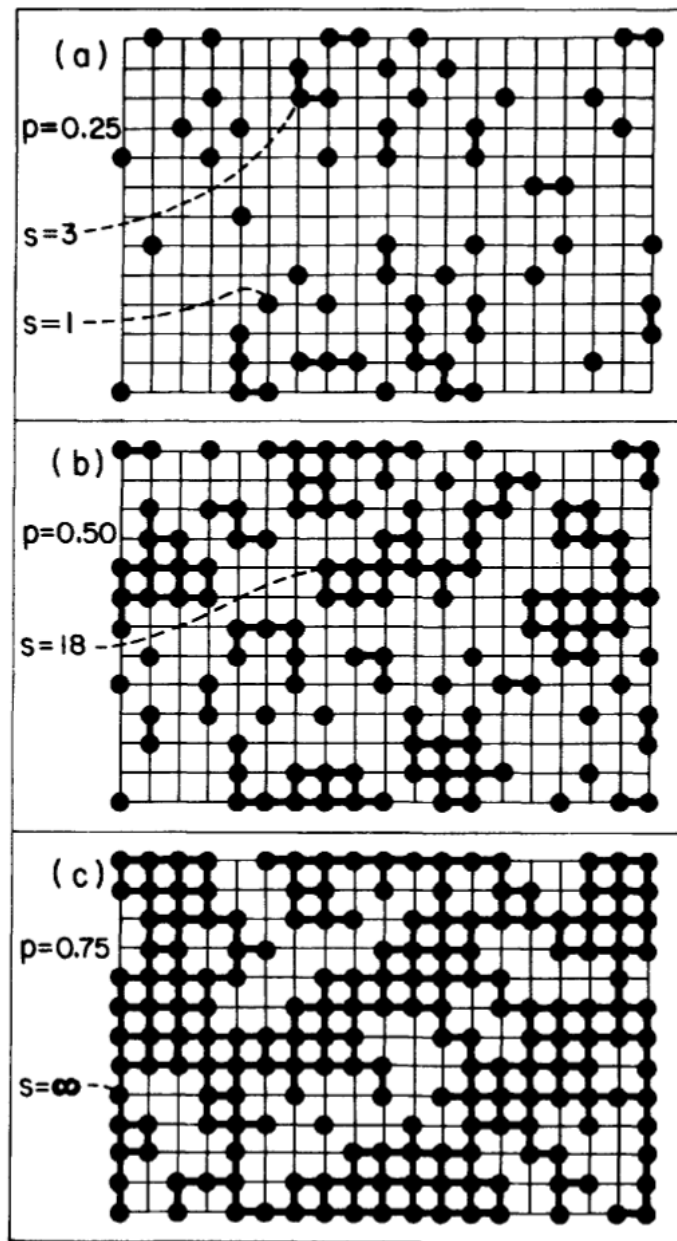
This figure, from page 137 of Zallen, describes the problem in a 2D square mesh.

At some precise critical number of random snips, current flow stops.

This is an example of bond percolation as opposed to site percolation.



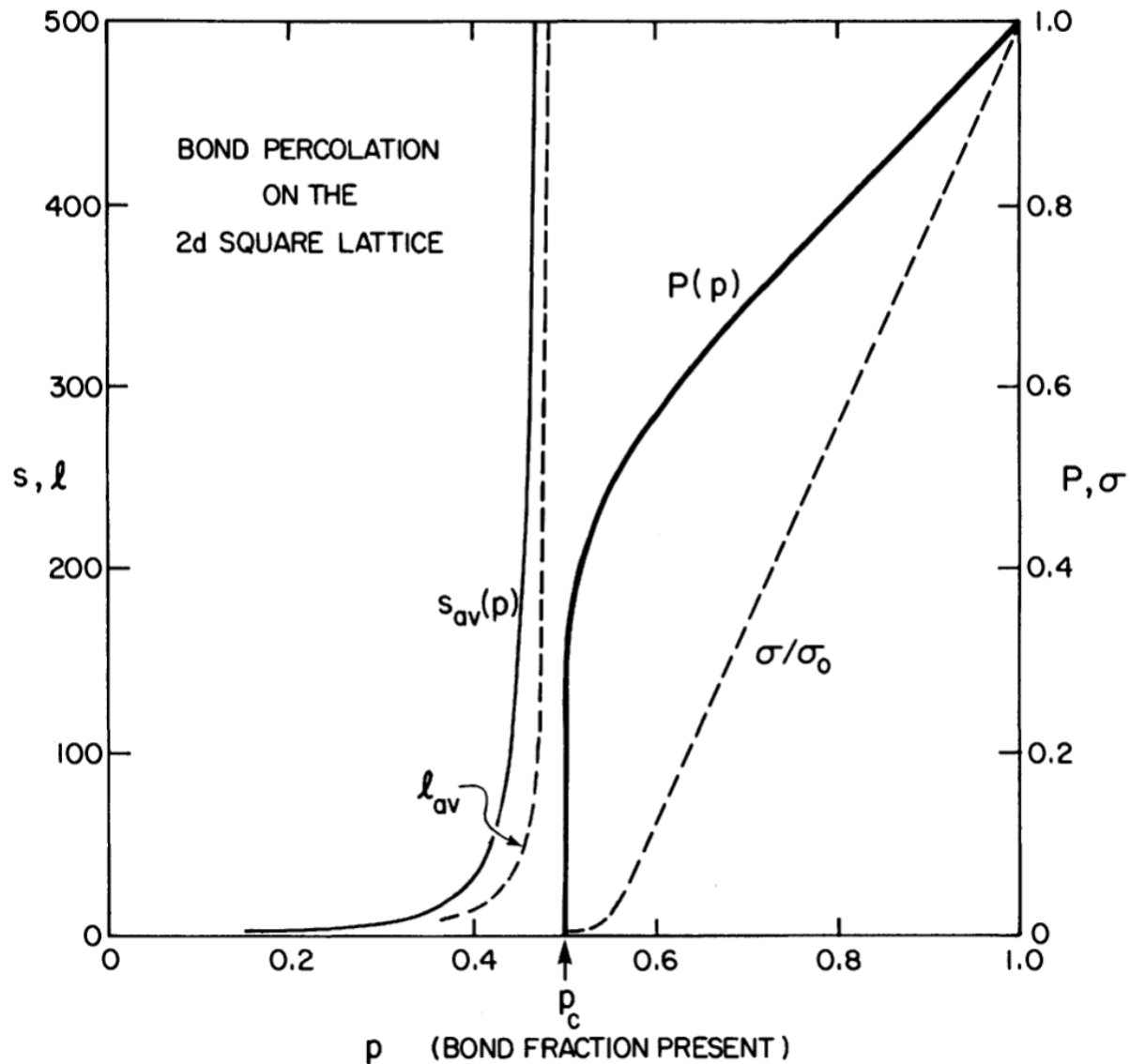
R. Zallen, *The Physics of Amorphous Solids*, Wiley-VCH, 2004.



This figure is from page 143 of Zallen.

This shows site percolation on a square lattice, with different site filling fractions p . For $p = 0.75$ in (c), the cluster formed by connecting neighboring atoms spans the whole lattice, and a percolation path is created.

R. Zallen, *The Physics of Amorphous Solids*, Wiley-VCH, 2004.



This figure is from page 146 of Zallen.

Computer simulations on a large square lattice; $s_{av}(p)$ is the average cluster size, and $P(p)$ is the percolation probability.

R. Zallen, *The Physics of Amorphous Solids*, Wiley-VCH, 2004.

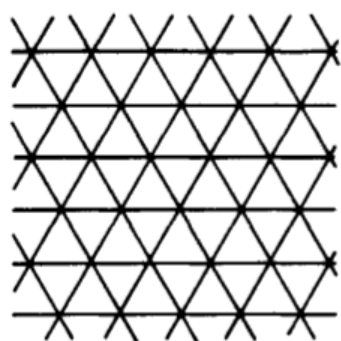
Where is percolation applicable?

<i>Phenomenon or System</i>	<i>Transition</i>
Flow of liquid in a porous medium	Local/extended wetting
Spread of disease in a population	Containment/epidemic
Communication or resistor networks	Disconnected/connected
Conductor-insulator composite materials	Insulator/metal
Composite superconductor-metal materials	Normal/superconducting
Discontinuous metal films	Insulator/metal
Stochastic star formation in spiral galaxies	Nonpropagation/propagation
Quarks in nuclear matter	Confinement/nonconfinement
Thin helium films on surfaces	Normal/superfluid
Metal-atom dispersions in insulators	Insulator/metal
Dilute magnets	Para/ferromagnetic
Polymer gelation, vulcanization	Liquid/gel
The glass transition	Liquid/glass
Mobility edge in amorphous semiconductors	Localized/extended states
Variable-range hopping in amorphous semiconductors	Resistor-network analog

This table is from page 148 of Zallen.

R. Zallen, *The Physics of Amorphous Solids*, Wiley-VCH, 2004.

Different lattices:

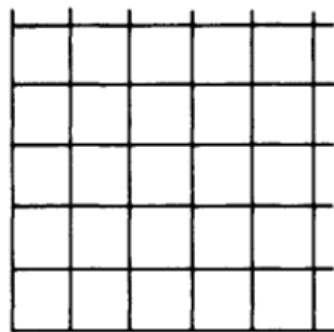


TRIANGULAR

$$z = 6$$

$$p_c^{\text{BOND}} = 0.3473$$

$$p_c^{\text{SITE}} = 0.5000$$

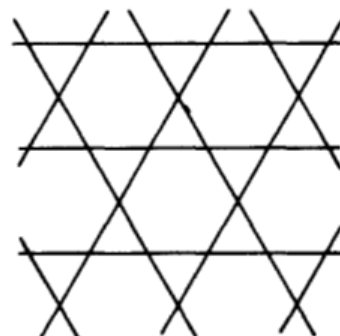


SQUARE

$$z = 4$$

$$p_c^{\text{BOND}} = 0.5000$$

$$p_c^{\text{SITE}} = 0.59$$

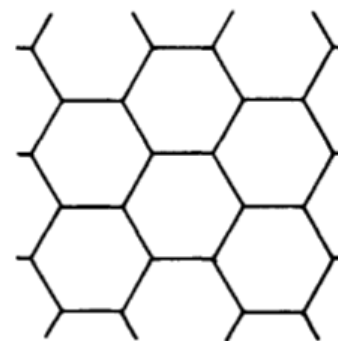


KAGOMÉ

$$z = 4$$

$$p_c^{\text{BOND}} = 0.45$$

$$p_c^{\text{SITE}} = 0.6527$$

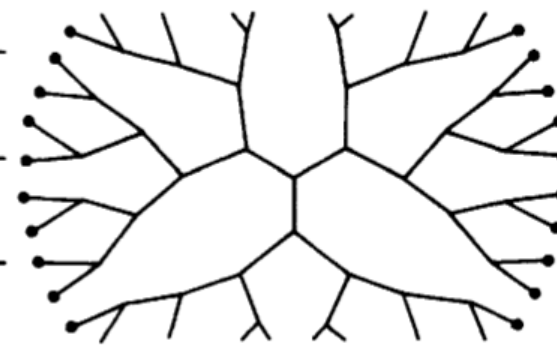


HONEYCOMB

$$z = 3$$

$$p_c^{\text{BOND}} = 0.6527$$

$$p_c^{\text{SITE}} = 0.70$$



BETHE LATTICE

$$z = 3$$

$$p_c^{\text{BOND}} = 0.5000$$

$$p_c^{\text{SITE}} = 0.5000$$

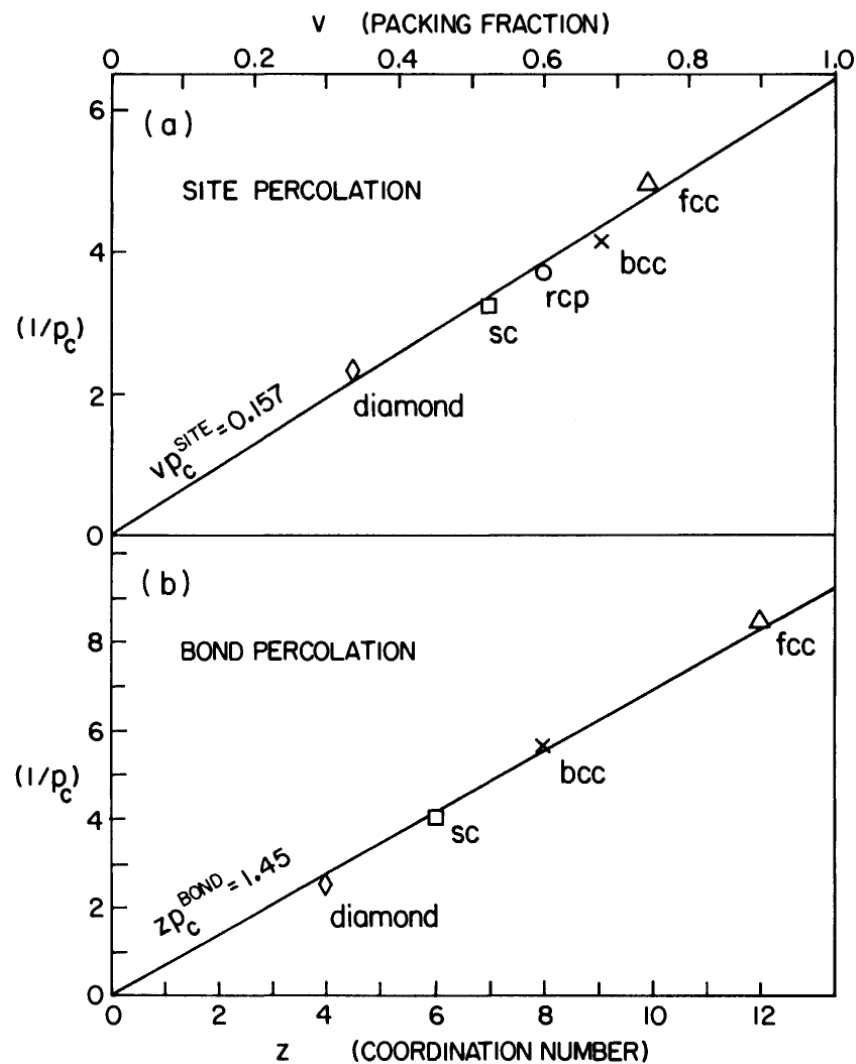
This table is from page 168 of Zallen.

R. Zallen, *The Physics of Amorphous Solids*, Wiley-VCH, 2004.

<i>Dimensionality</i> <i>d</i>	<i>Lattice or Structure</i>	p_c^{bond}	p_c^{site}	<i>Coordination</i> <i>z</i>	<i>Filling Factor</i> <i>v</i>	zp_c^{bond}	$vp_c^{site} \equiv \phi_c$
1	Chain	1	1	2	1	2	1
2	Triangular	0.3473	0.5000	6	0.9069	2.08	0.45
2	Square	0.5000	0.593	4	0.7854	2.00	0.47
2	Kagomé	0.45	0.6527	4	0.6802	1.80	0.44
2	Honeycomb	0.6527	0.698	3	0.6046	1.96	0.42
						2.0 ± 0.2	0.45 ± 0.03
3	fcc	0.119	0.198	12	0.7405	1.43	0.147
3	bcc	0.179	0.245	8	0.6802	1.43	0.167
3	sc	0.247	0.311	6	0.5236	1.48	0.163
3	Diamond	0.388	0.428	4	0.3401	1.55	0.146
3	rcp		$[0.27]^b$		$0.637 [0.6]^b$		$[0.16]^b$
						1.5 ± 0.1	0.16 ± 0.02
4	sc	0.160	0.197	8	0.3084	1.3	0.061
4	fcc		0.098	24	0.6169		0.060
5	sc	0.118	0.141	10	0.1645	1.2	0.023
5	fcc		0.054	40	0.4653		0.025
6	sc	0.094	0.107	12	0.0807	1.1	0.009

Page 170 of Zallen.

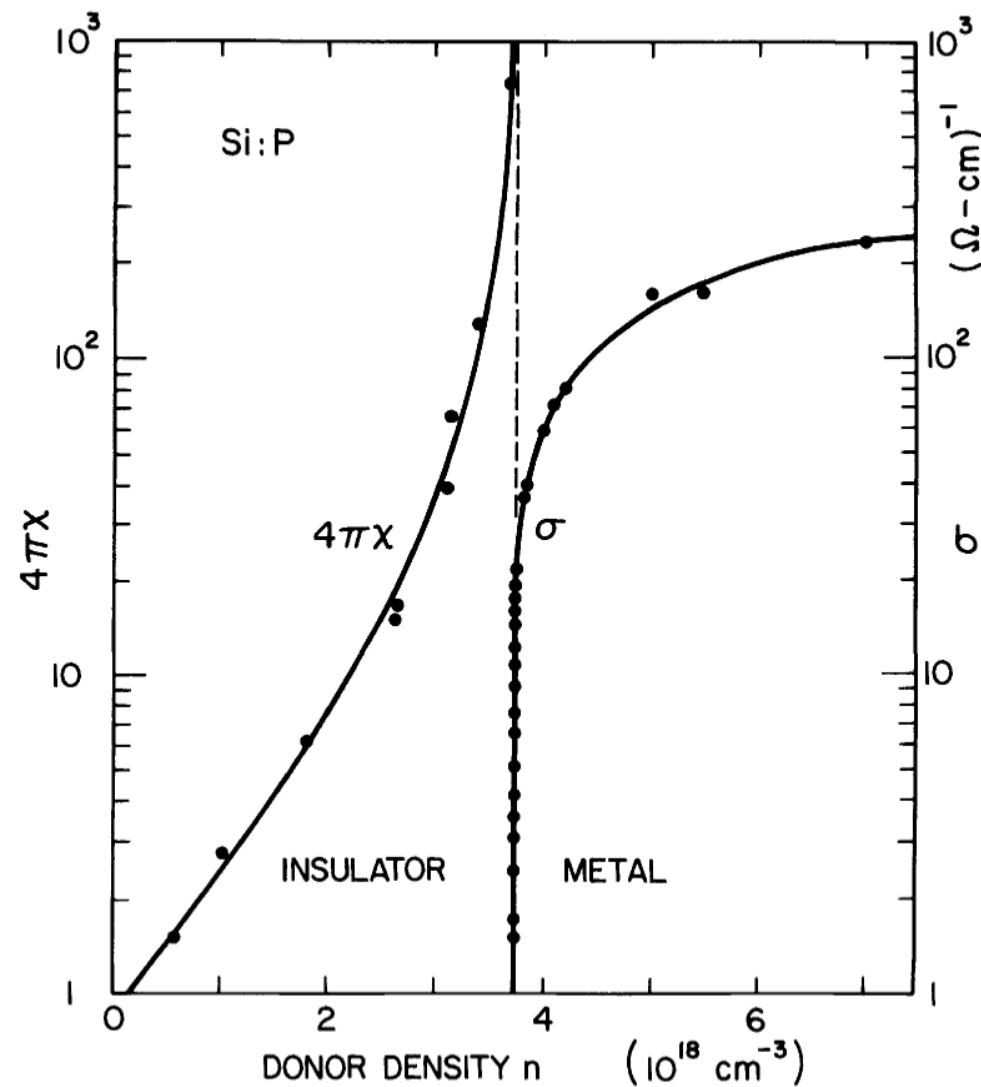
R. Zallen, The Physics of Amorphous Solids, Wiley-VCH, 2004.



This figure is from page 187 of Zallen.

There is a simple scaling in 3D, between both the site and bond percolation thresholds, with the packing fraction and coordination number, and the percolation thresholds.

R. Zallen, *The Physics of Amorphous Solids*, Wiley-VCH, 2004.



This figure is from page 243 of Zallen.

The non-metal to metal transition on Si:P.

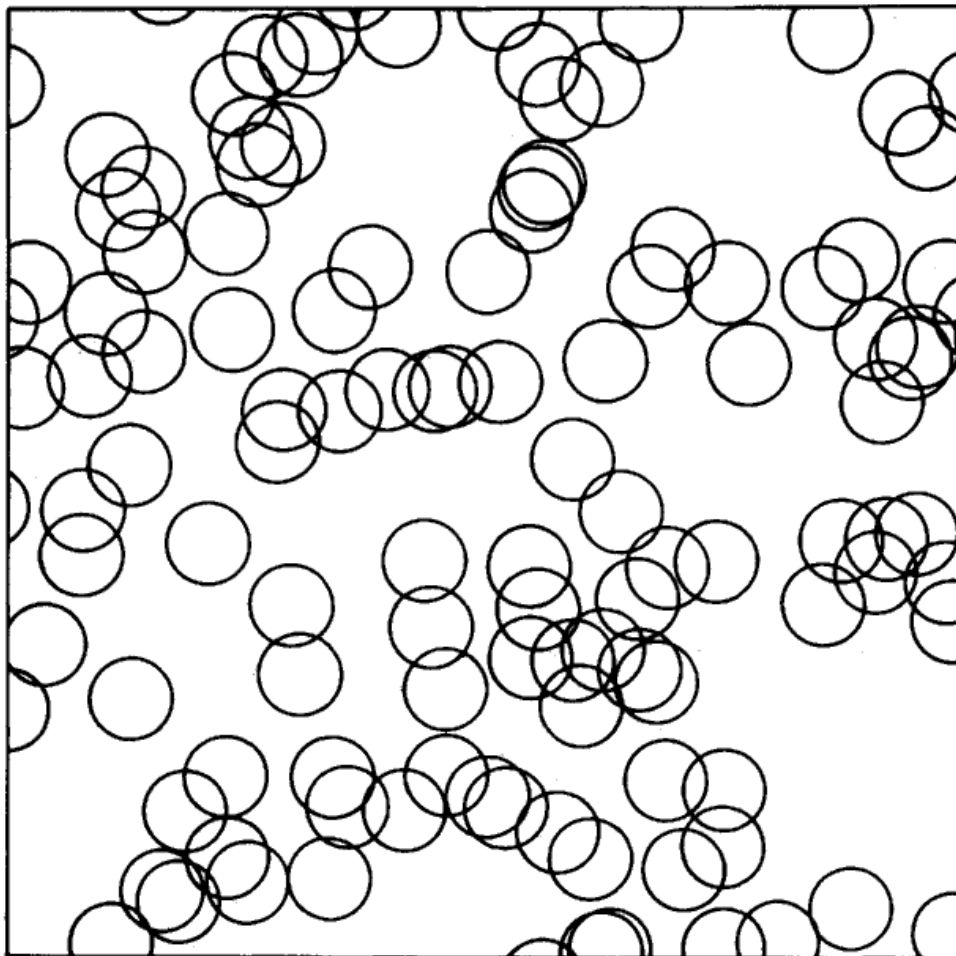
$4\pi\chi$ is the dielectric susceptibility.

Note the similarity with percolation (the third slide in this set of slides).

Measurements by Rosenbaum and others at 10 mK.

This is of the percolation, not of atoms but of Bohr radii.

R. Zallen, *The Physics of Amorphous Solids*, Wiley-VCH, 2004.

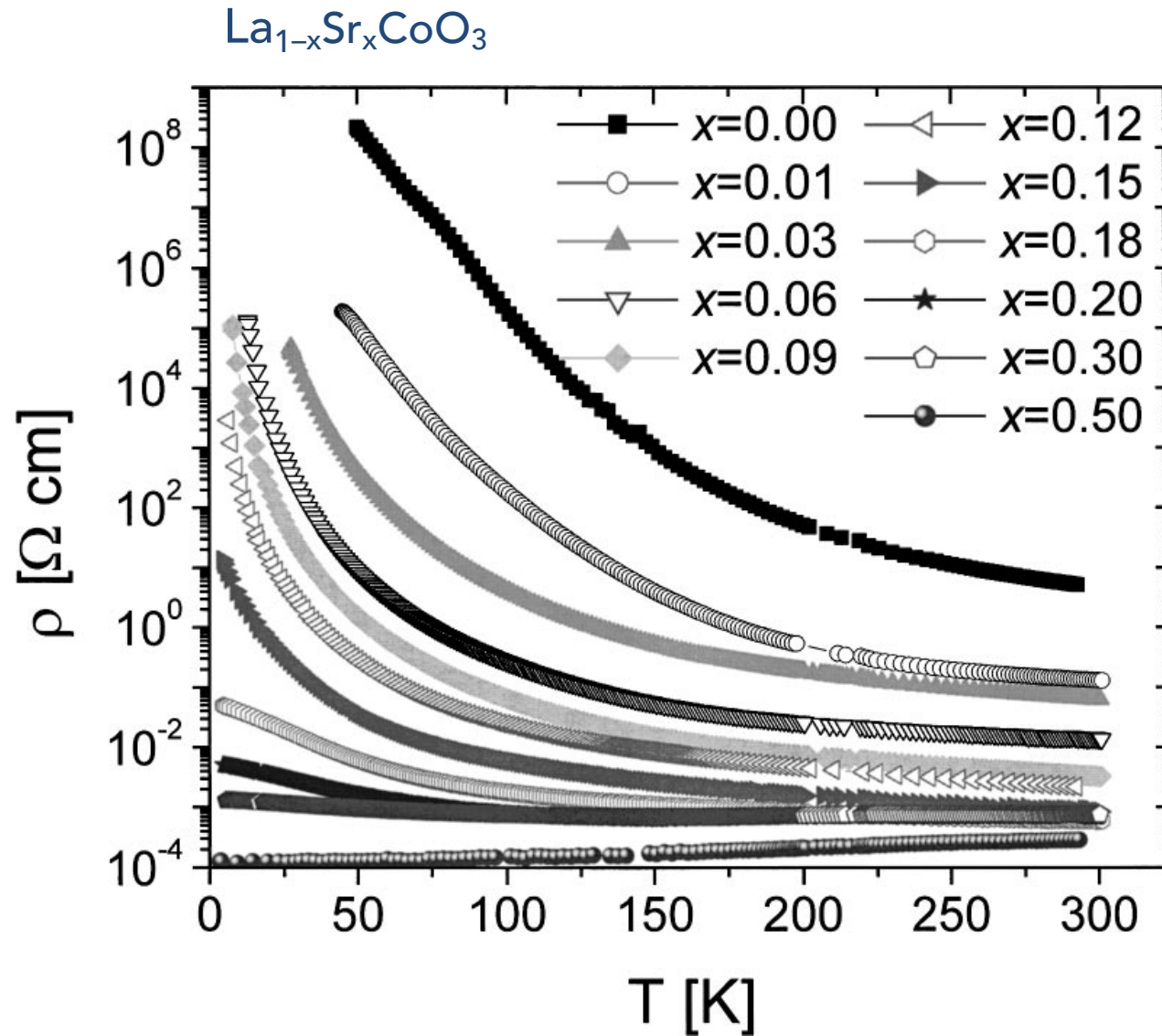


This figure is from page 244 of Zallen.

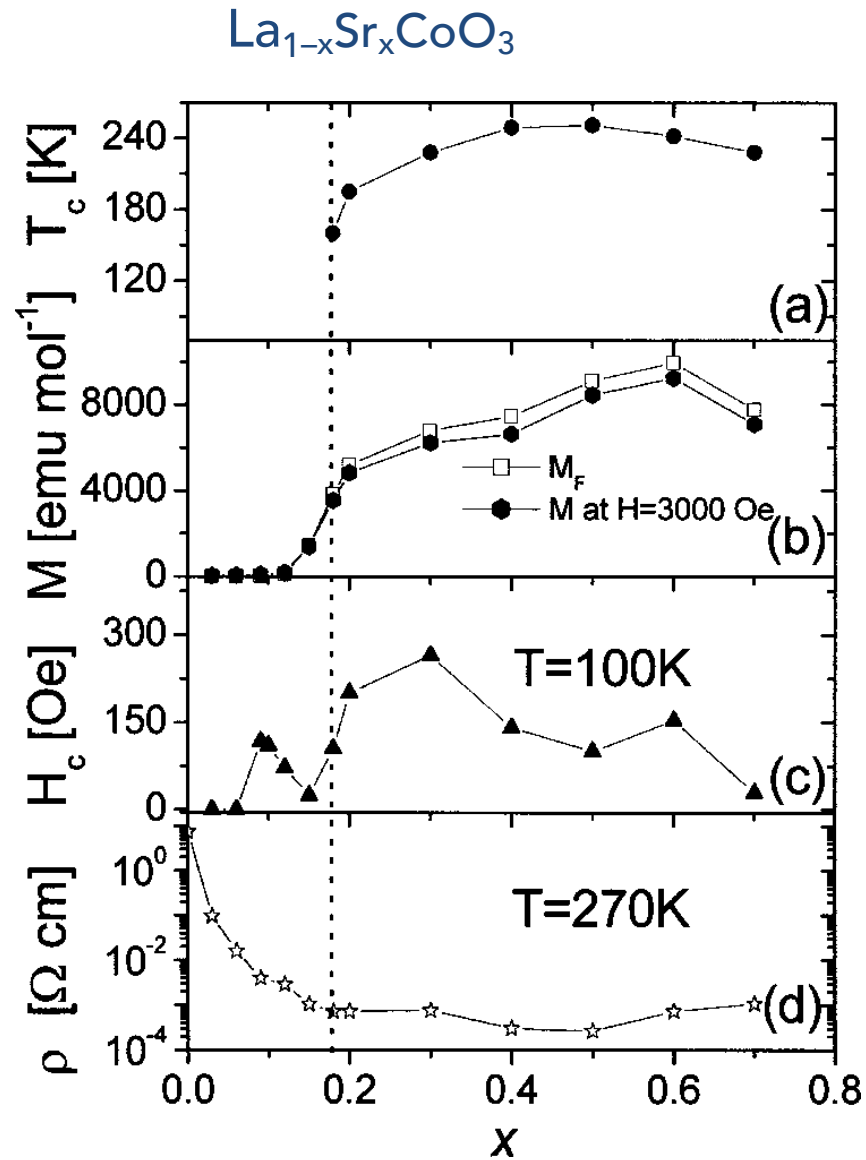
Showing the percolation of hydrogenic wavefunctions around the phosphorus donor atoms (much larger than the interatomic spacing).

Since P substitution is random, this is a problem of percolation in a random close packing.

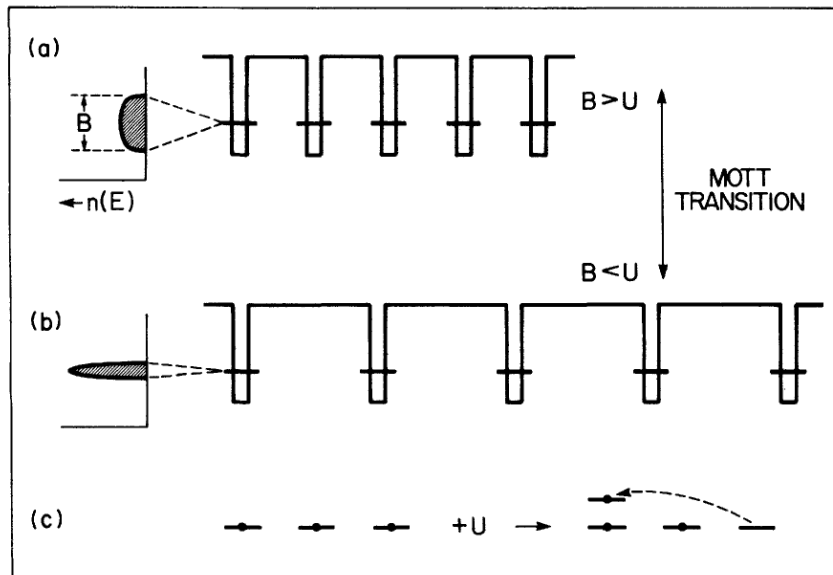
R. Zallen, *The Physics of Amorphous Solids*, Wiley-VCH, 2004.



Wu, Leighton, *Phys. Rev. B* 67 (2003) 174408.

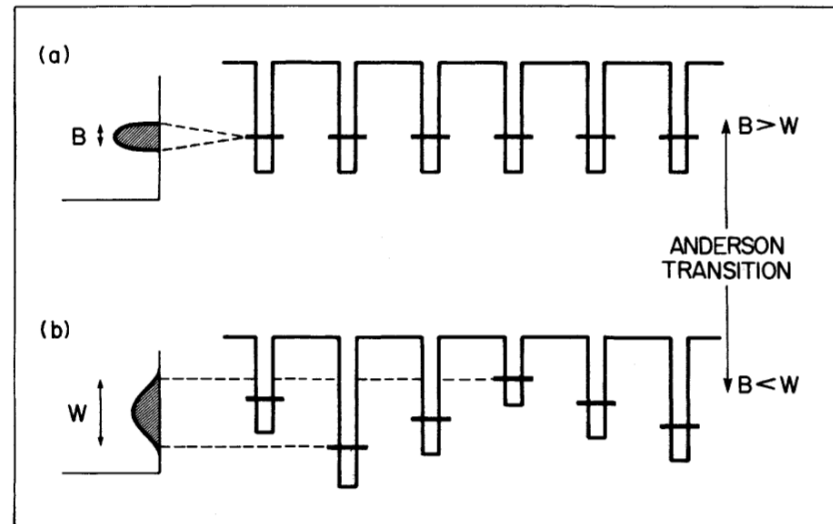


Wu, Leighton, *Phys. Rev. B* **67** (2003) 174408.

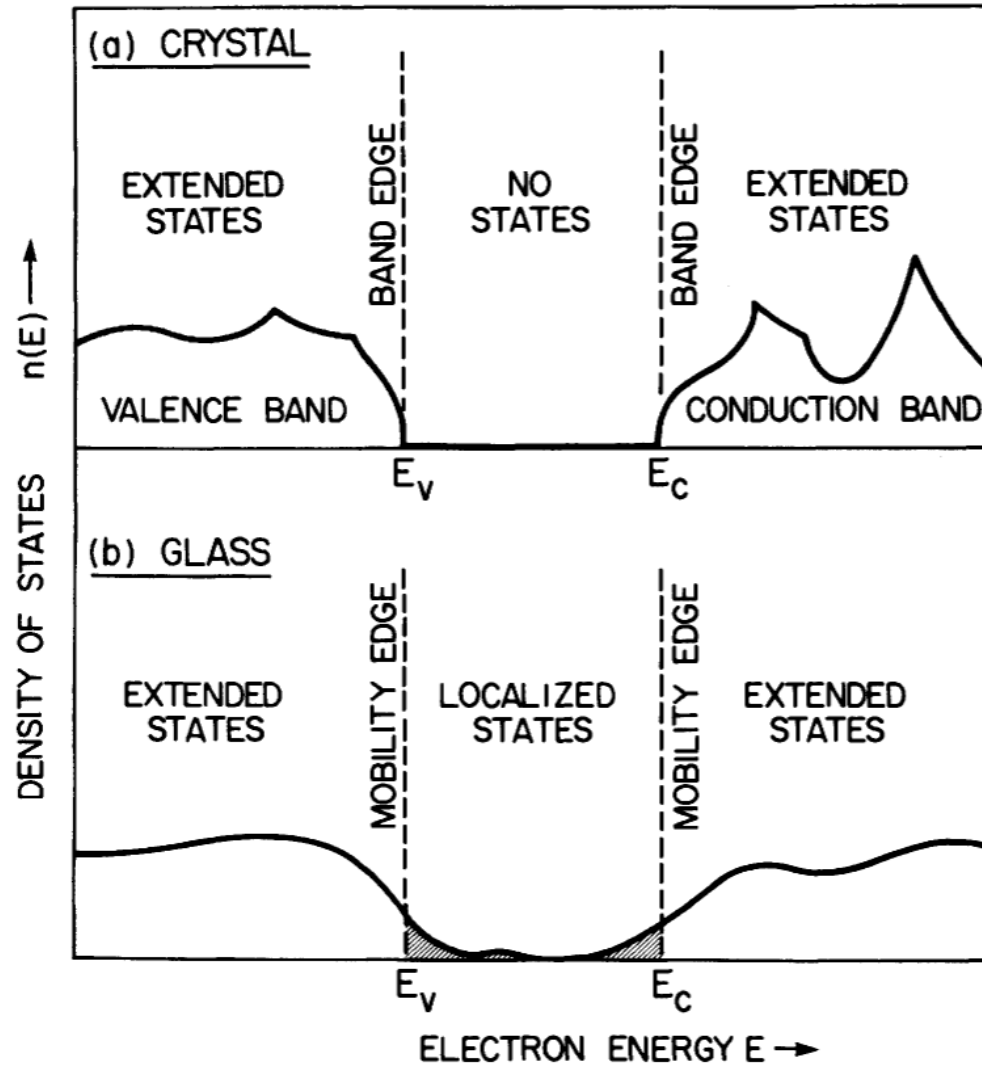


This figure is from pages 229 and 232 of Zallen.

The Mott and Anderson transitions represented graphically in 1D.



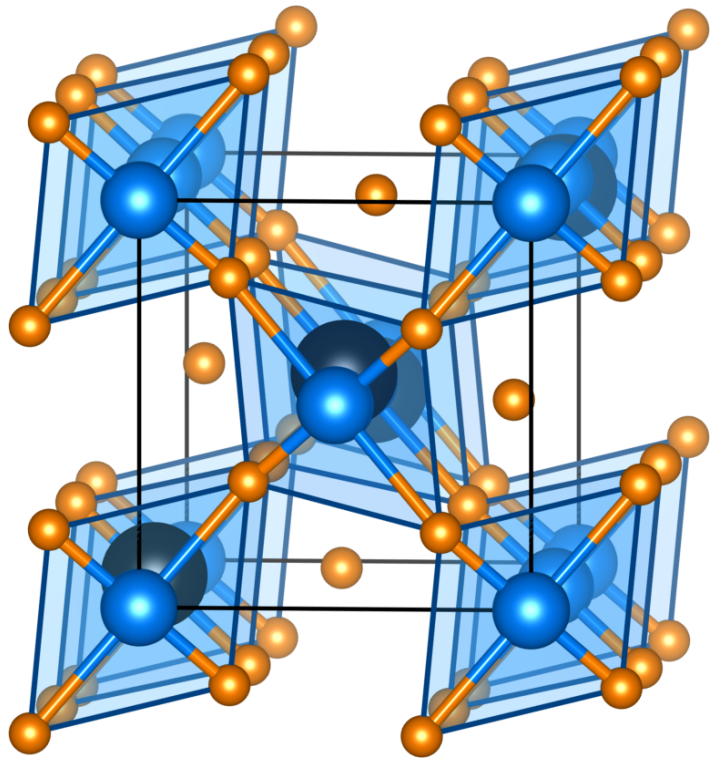
R. Zallen, *The Physics of Amorphous Solids*, Wiley-VCH, 2004.



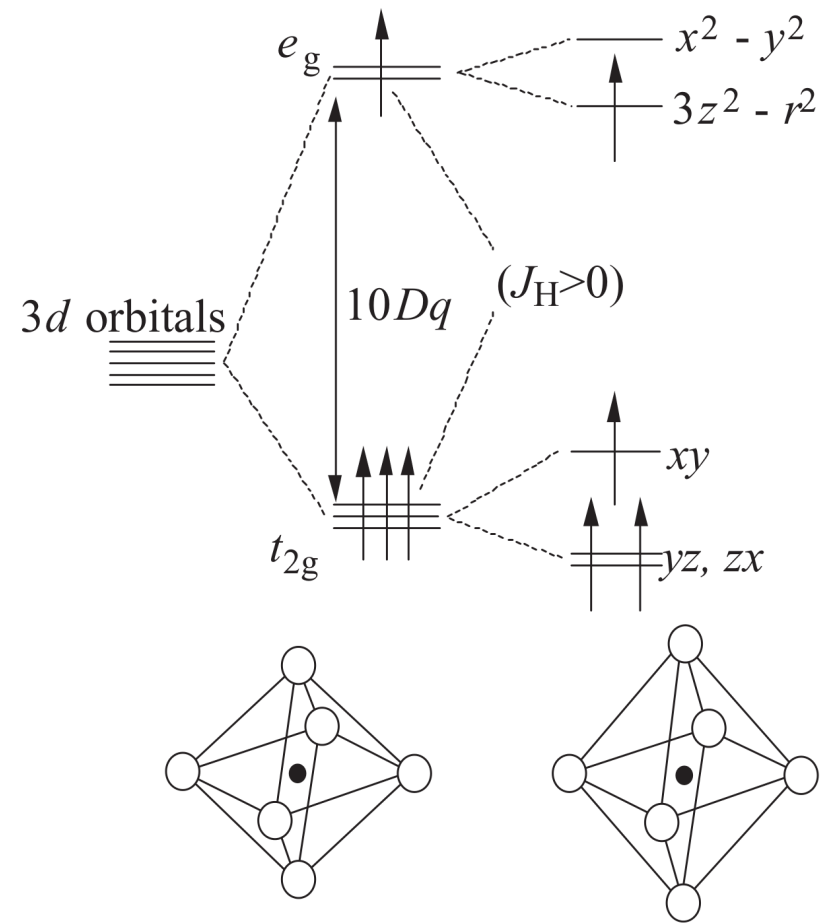
This figure is from page 235 of Zallen.

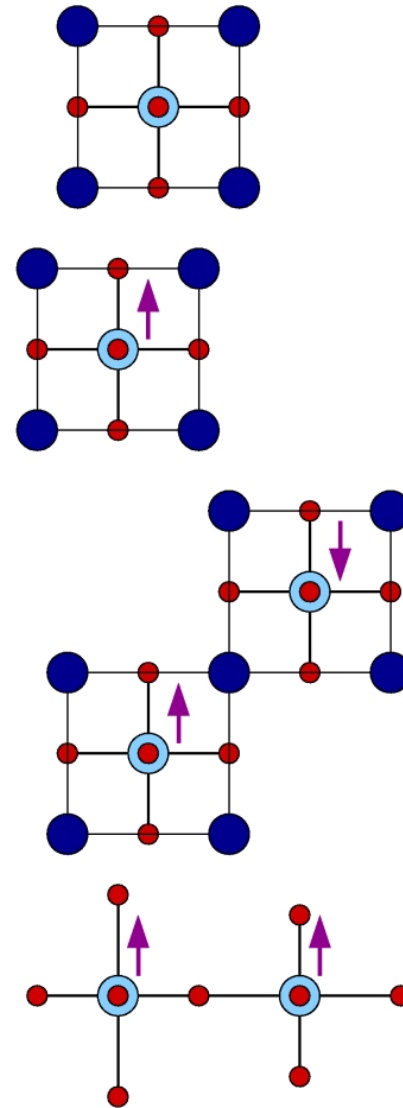
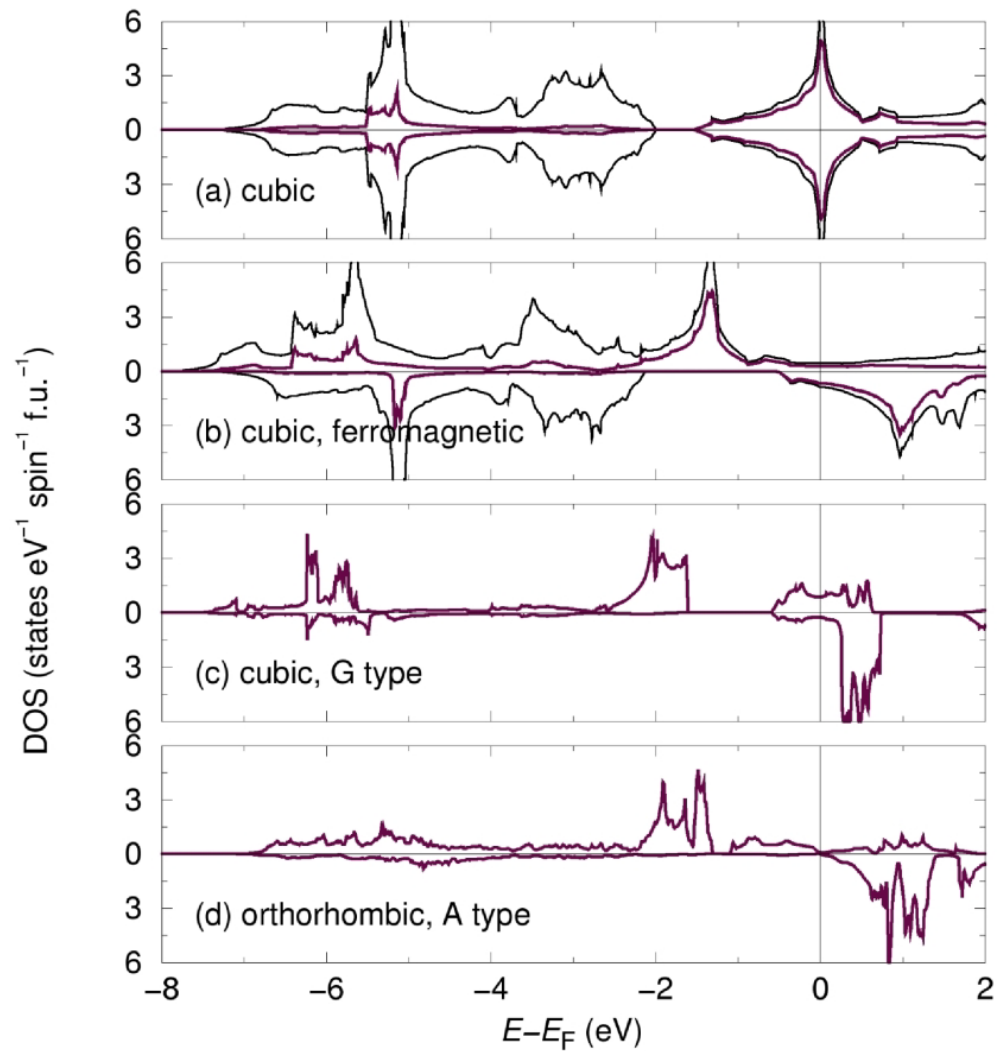
The notion of the mobility edge.

R. Zallen, *The Physics of Amorphous Solids*, Wiley-VCH, 2004.

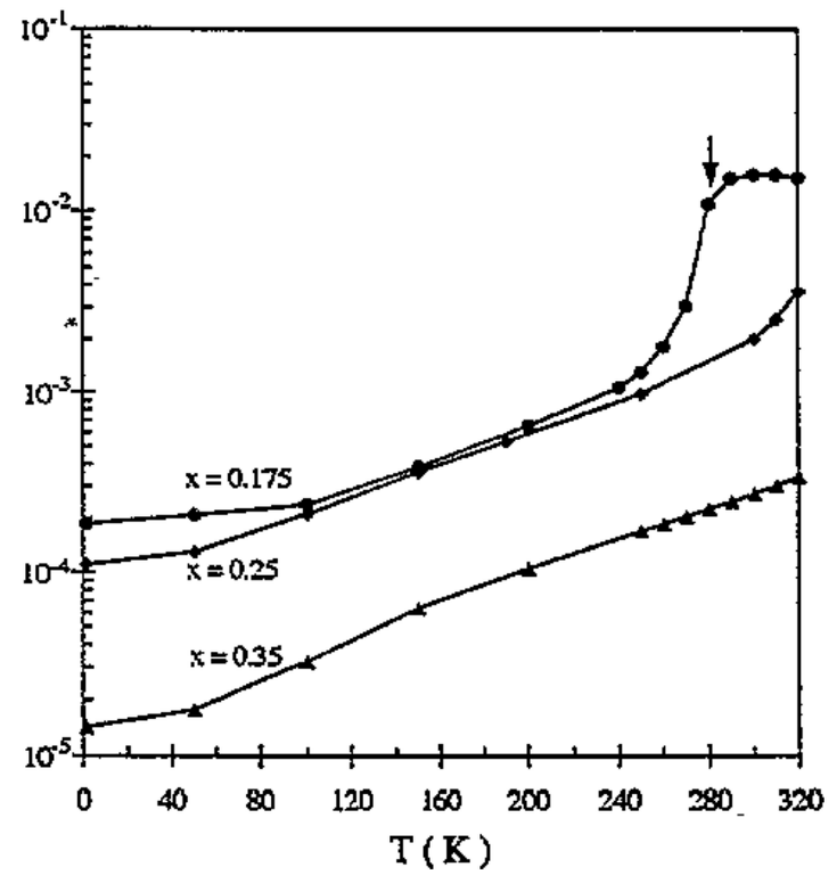
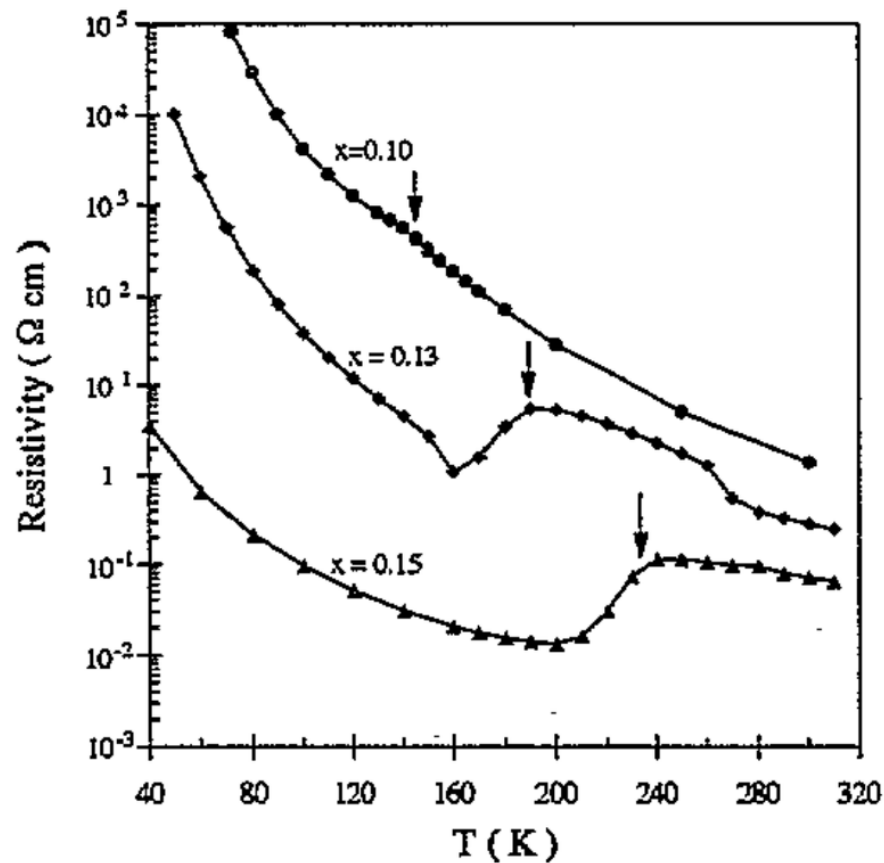


Jahn-Teller distorted
orthorhombic perovskite
(space group $Pnma$)



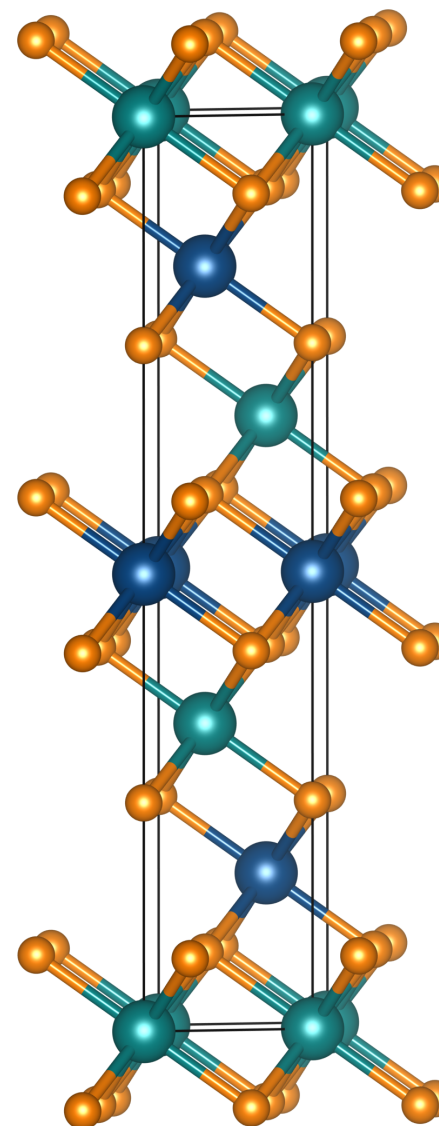
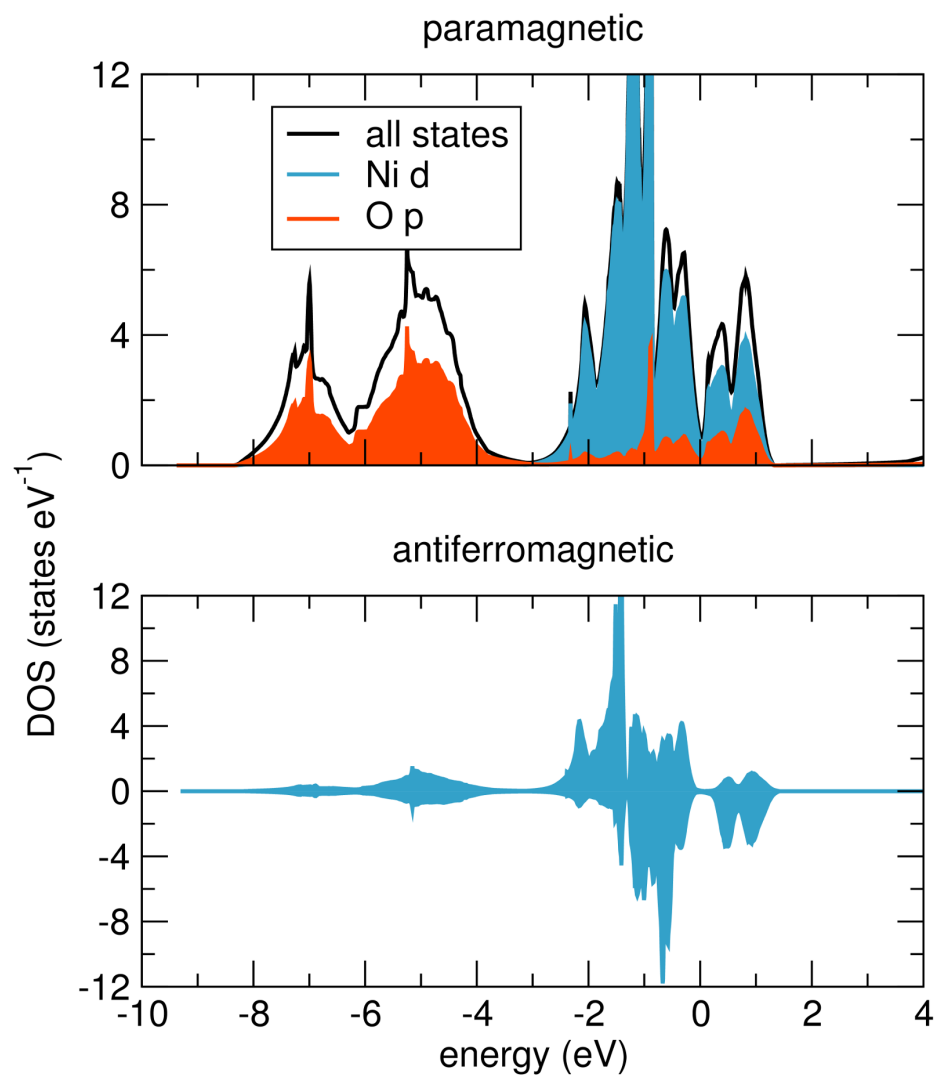


Structure and magnetism do not explain the insulating behavior.



Electrical resistivity behavior in $\text{La}_{1-x}\text{Sr}_x\text{MnO}_3$

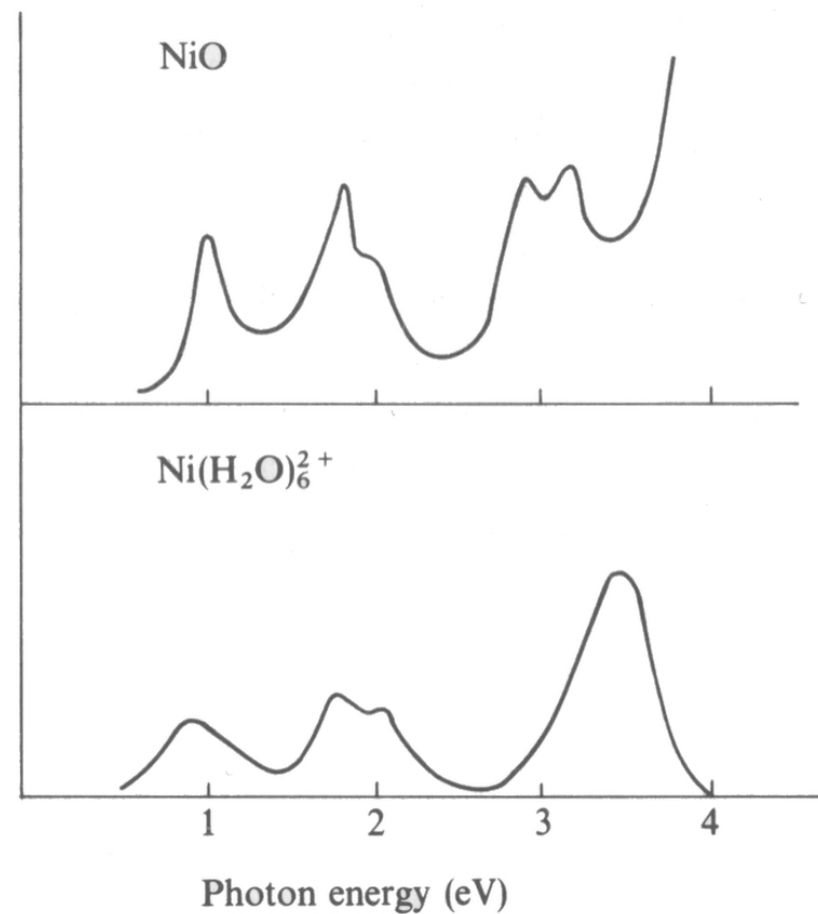
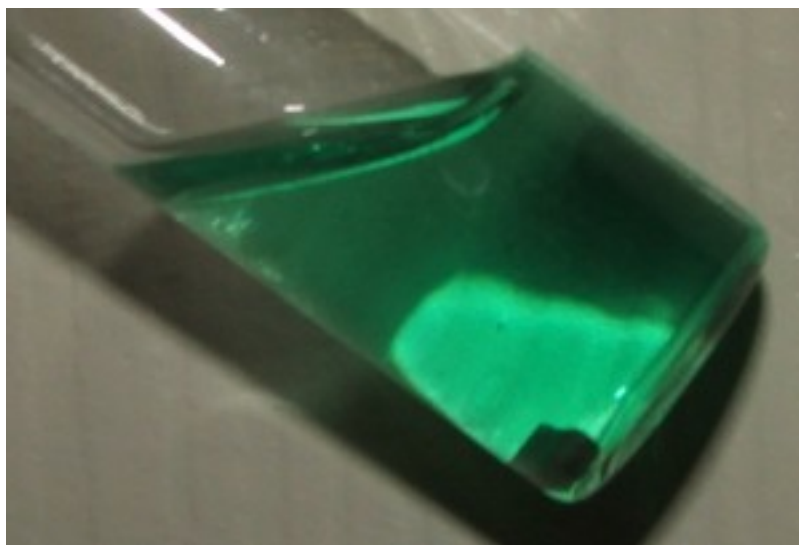
Anane, Dupast, Dang, Renard, Veillet, de Leon Guevare, Millot, Pinsard, Revcolevschi, *J. Phys.: Condens. Matter* 7 (1995) 7015-7021.



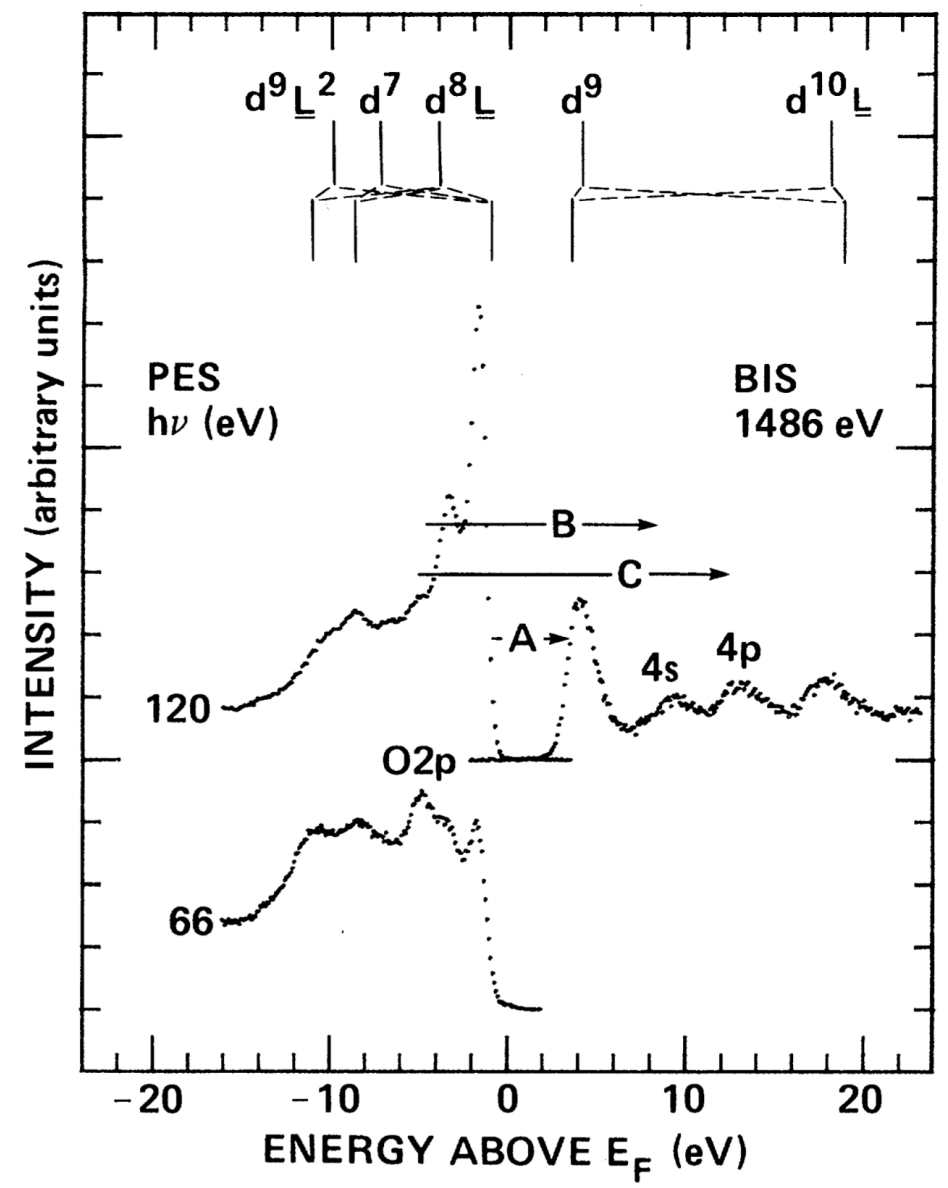
NiO displays the color of isolated Ni²⁺ in solution, with similar spectra.

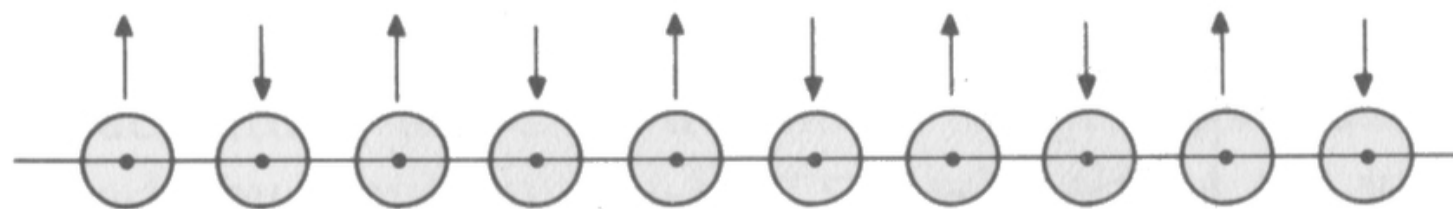
Correlation and the Hubbard model: NiO

NiO displays the color of isolated Ni^{2+} in solution, with similar

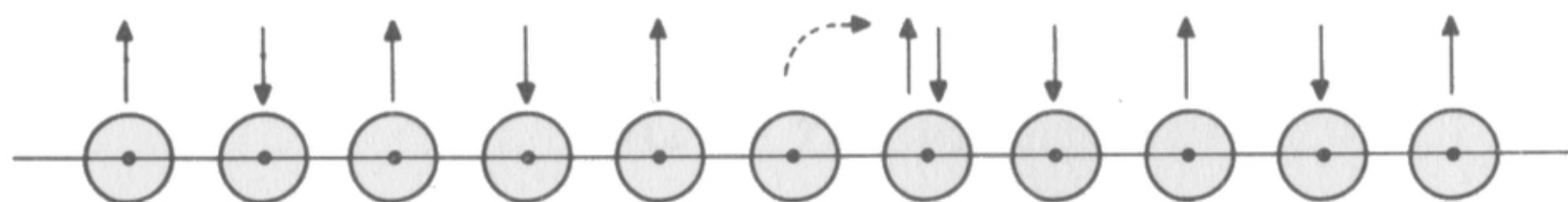


From the Cox The Electronic Structure and Chemistry of Solids (Oxford Science Publications) 1st Edition, page 151





(a)



(b)

From the Cox text, page 135

Consider a chain of orbitals, each with one electron. To hop an electron, an orbital has to be ionized at cost I , which is compensated a little by the electron affinity A .

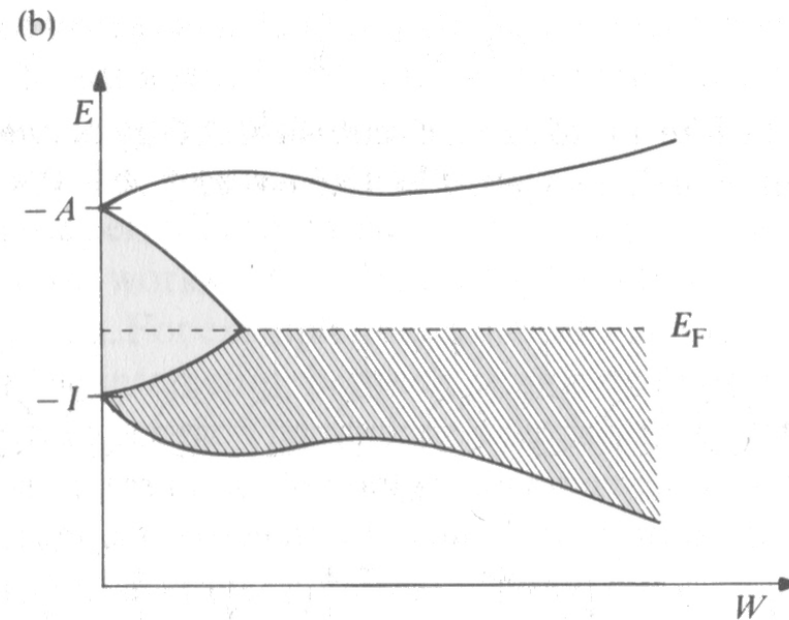
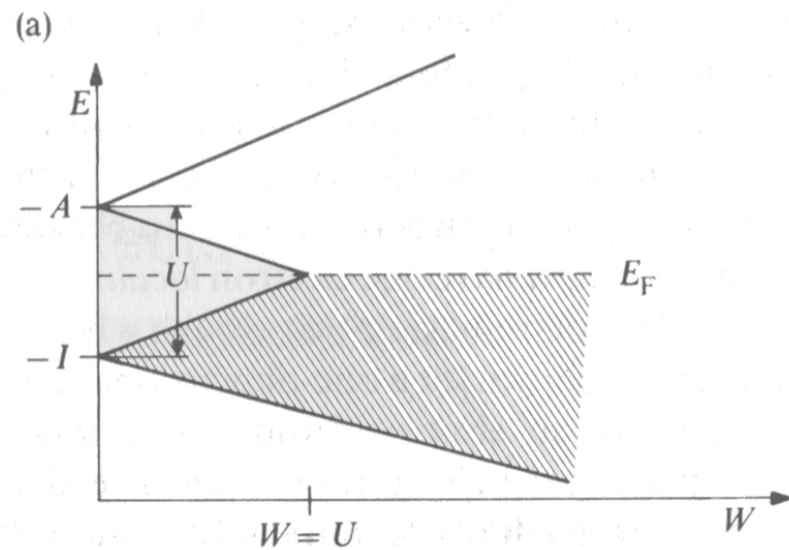
$$U = I - A$$

For H atoms, $I = 13.6$ eV and $A = 0.8$ eV, meaning $U = 12.8$ eV. However, this does not account for some screening (due to the dielectric not being vacuum).

$$H = -t \sum_{\langle i,j \rangle, \sigma} (c_{i,\sigma}^\dagger c_{j,\sigma} + c_{j,\sigma}^\dagger c_{i,\sigma}) + U \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow}$$

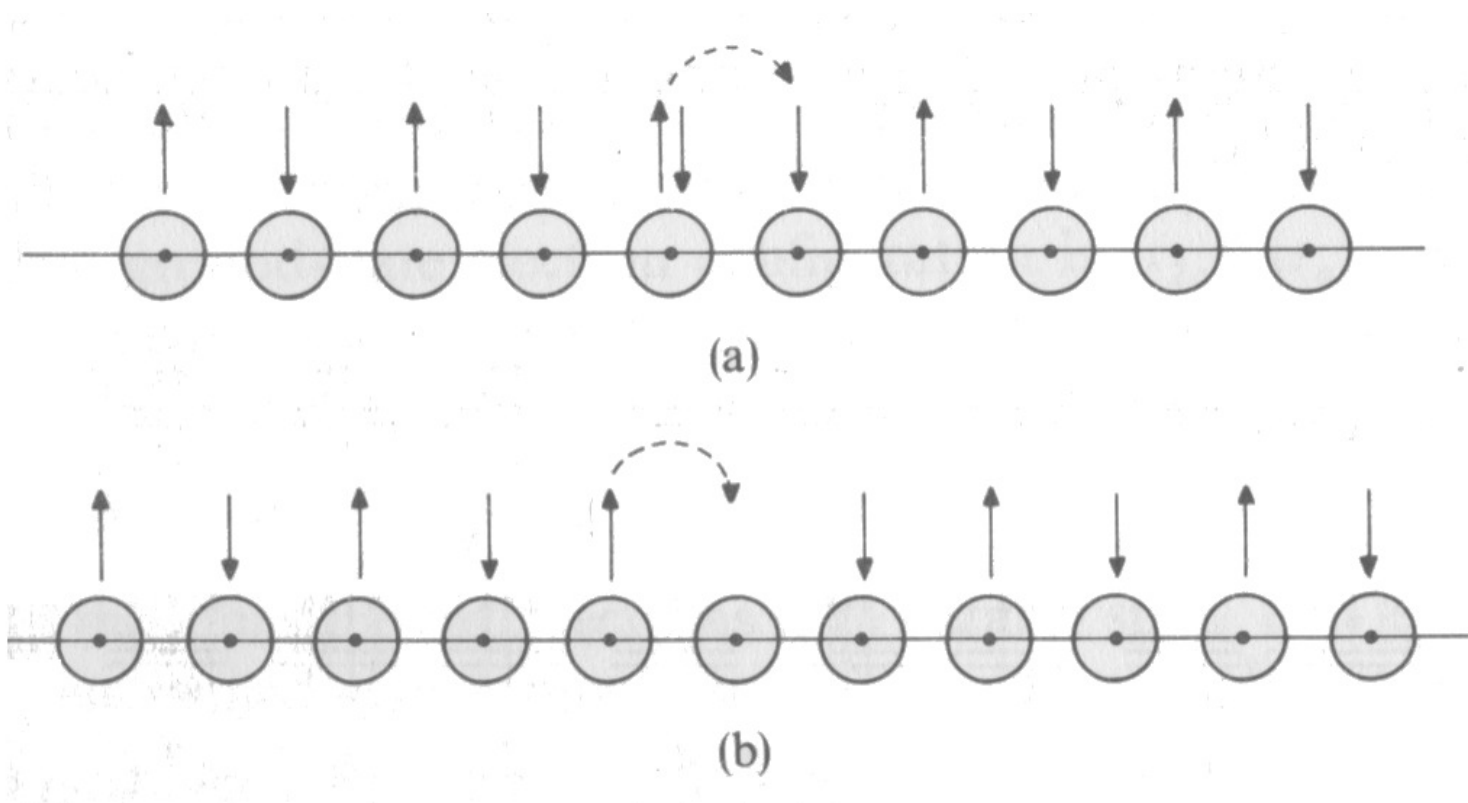
hopping or tight-binding (LCAO) part

double-
occupancy cost
or on-site
repulsion



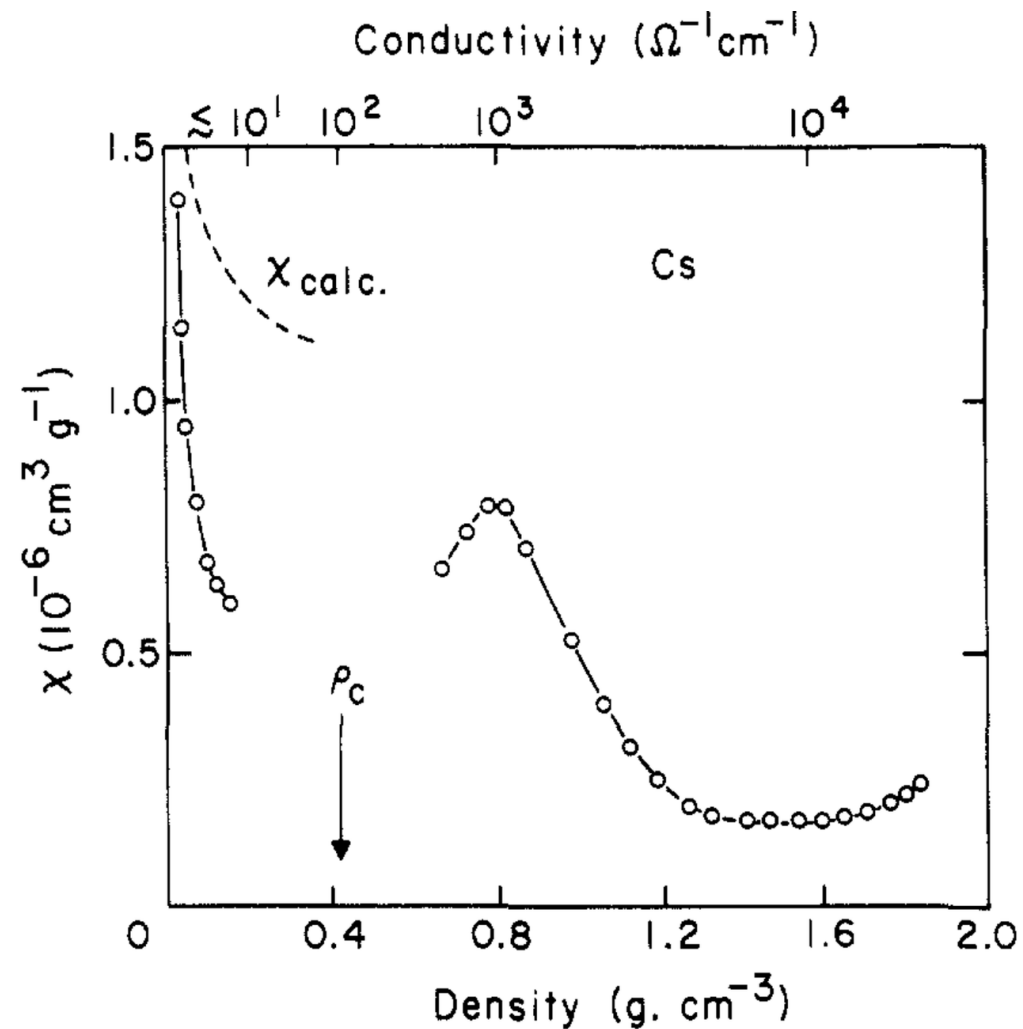
As the bandwidth is increased, (or as the atoms approach closer) the gap can close.

From the Cox text, page 137



Doping of holes (removal of electrons) as in (b) makes hopping much easier, with the on-site repulsion having been removed.

From the Cox text, page 149



On the insulating side of the M-I transition, magnetism of some sort (usually antiferromagnetism) manifests.

In the limit that $U \gg t$, the Hubbard model:

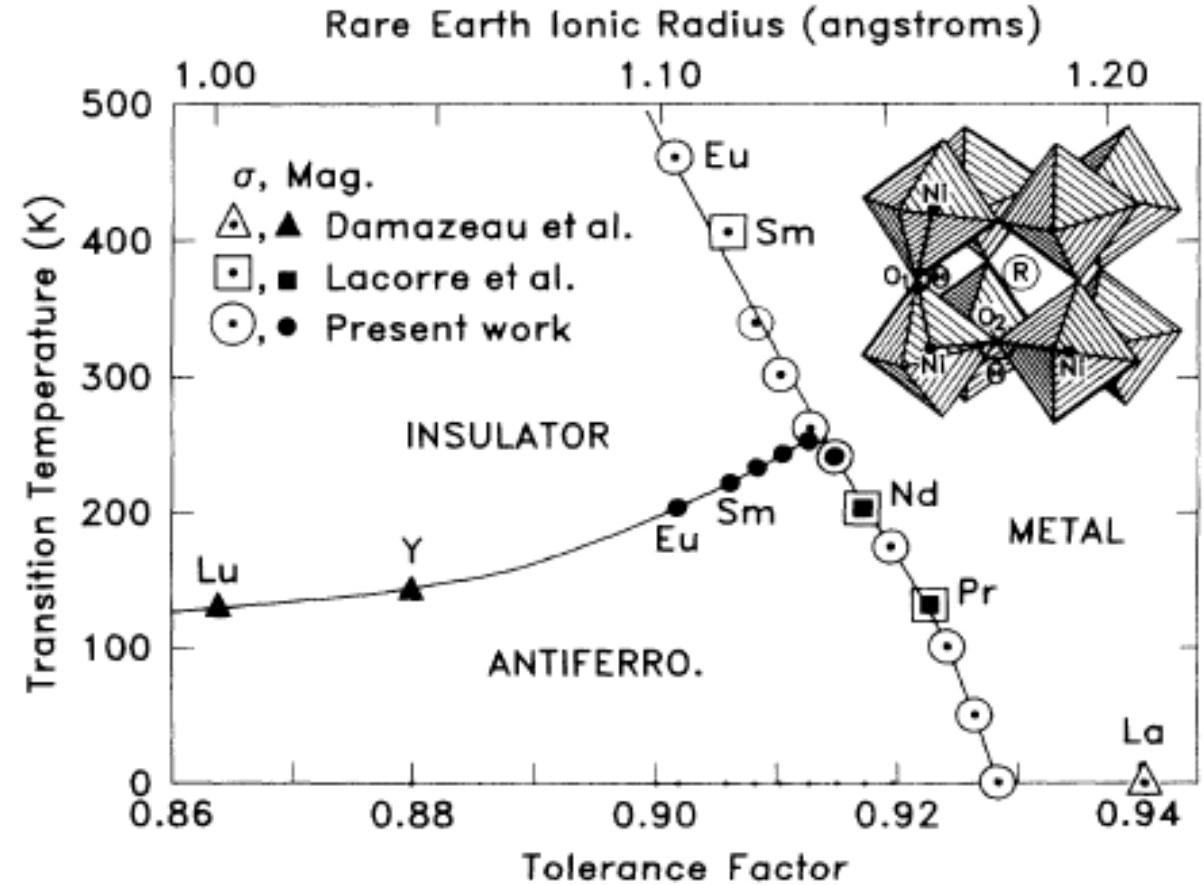
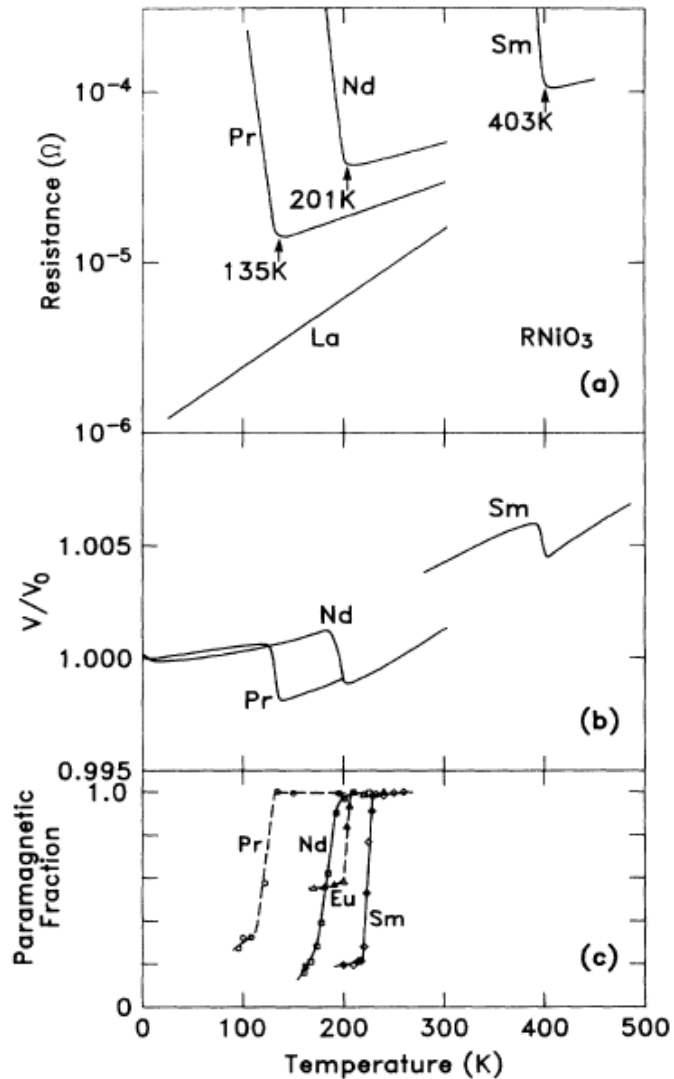
$$H = -t \sum_{\langle i,j \rangle, \sigma} (c_{i,\sigma}^\dagger c_{j,\sigma} + c_{j,\sigma}^\dagger c_{i,\sigma}) + U \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow}$$

Can be reduced to the Heisenberg Hamiltonian:

$$H = J \sum_{\langle ij \rangle} \vec{S}_1 \vec{S}_2 \quad \text{where} \quad J = \frac{4t^2}{U}$$

Correlation and the Hubbard model: The rare-earth nickel oxide perovskites

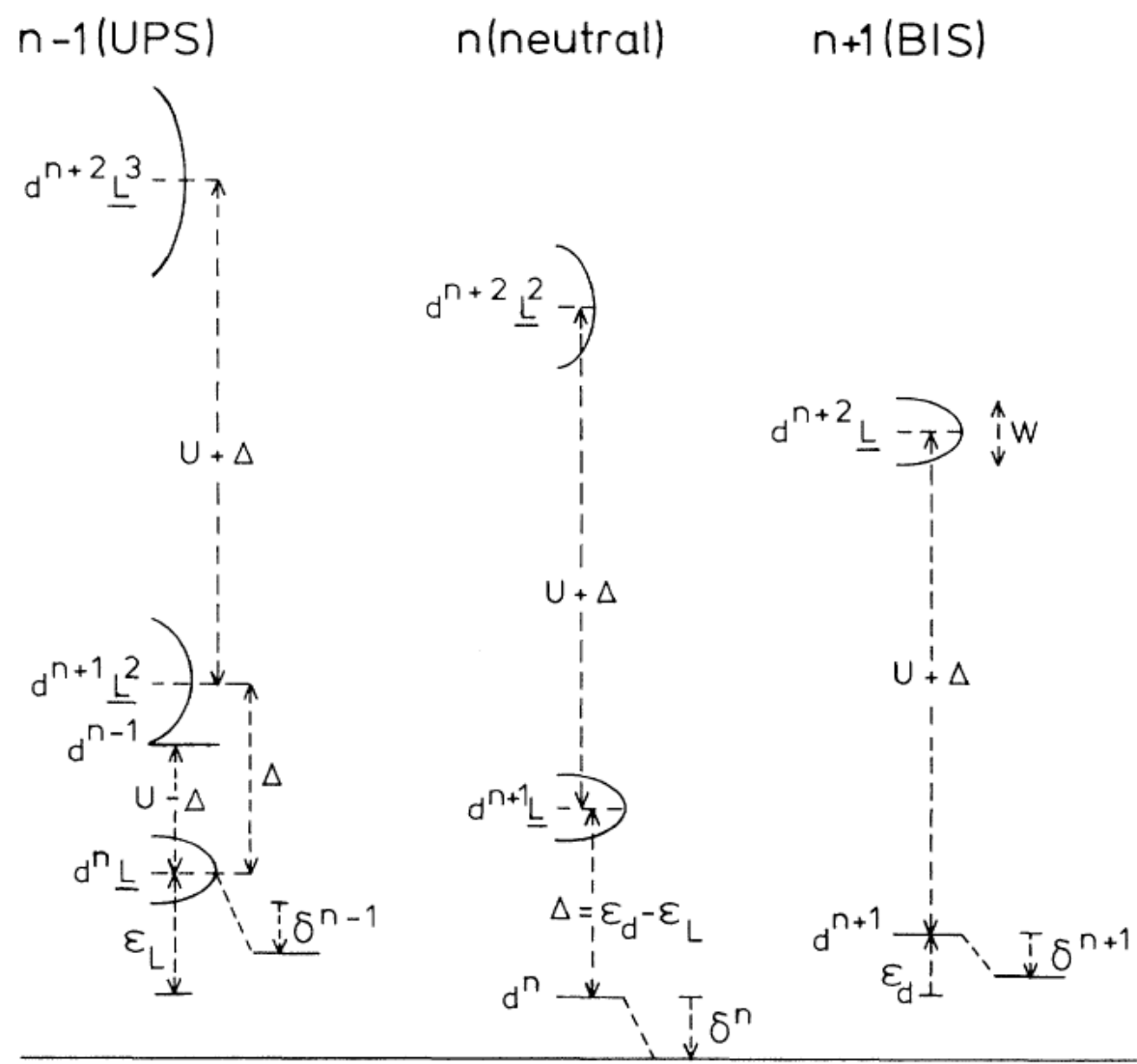
The example of LnNiO_3 , $\text{Ln} = \text{La, Pr, Nd, Sm}$:



Torrance, Lacorre, Nazzari, Ansaldo, Niedermayer, *Phys. Rev. B.* 45 (1992) 8209–8212.

The Zaanen-Sawatzky-Allen phase diagram

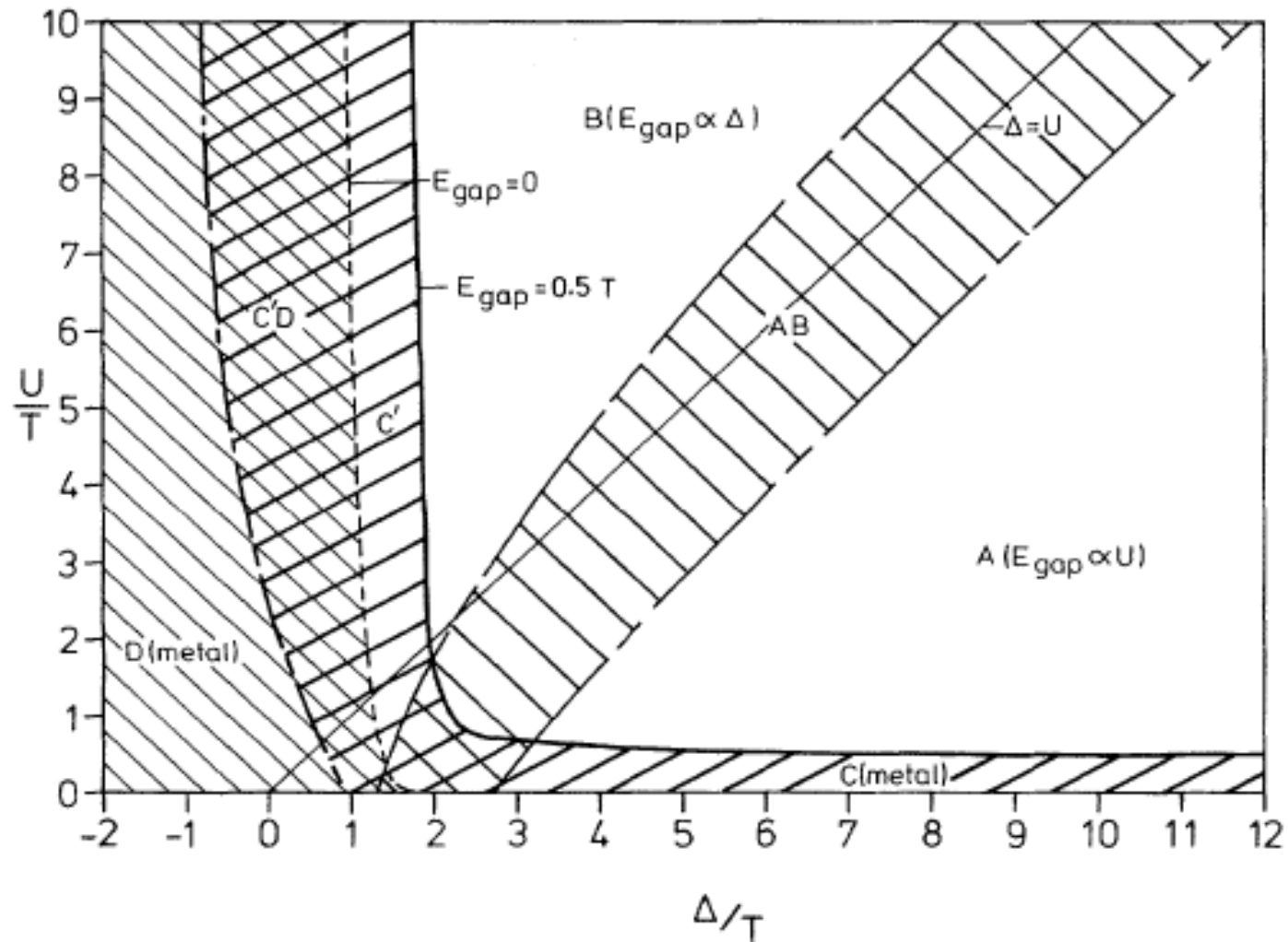
The position of transition metal d-states vs. anion p states:



Zaanen, Sawatzky, Allen, *Phys. Rev. Lett.* **55** (1985) 418–421.

The Zaanen-Sawatzky-Allen phase diagram

The actual diagram. Note that T is what we have been calling W , and W is proportional to t in the Hubbard Model.

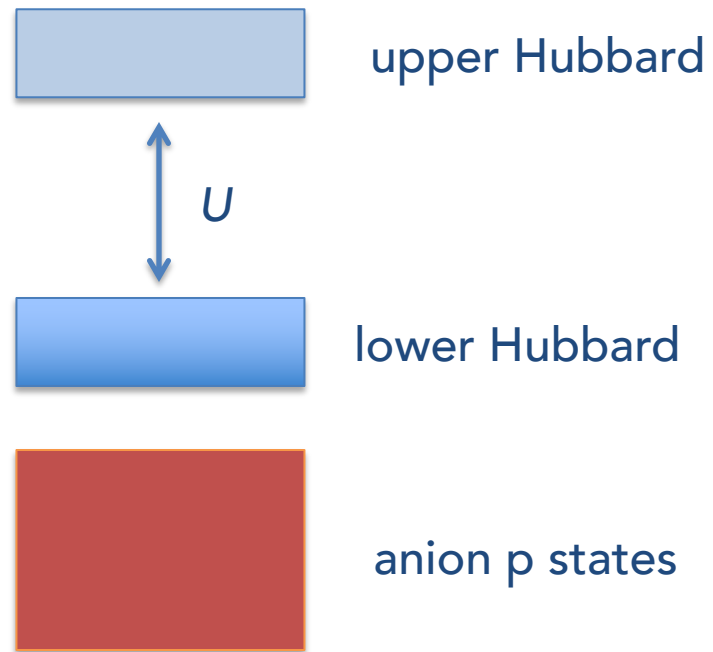


Zaanen, Sawatzky, Allen, *Phys. Rev. Lett.* **55** (1985) 418–421.

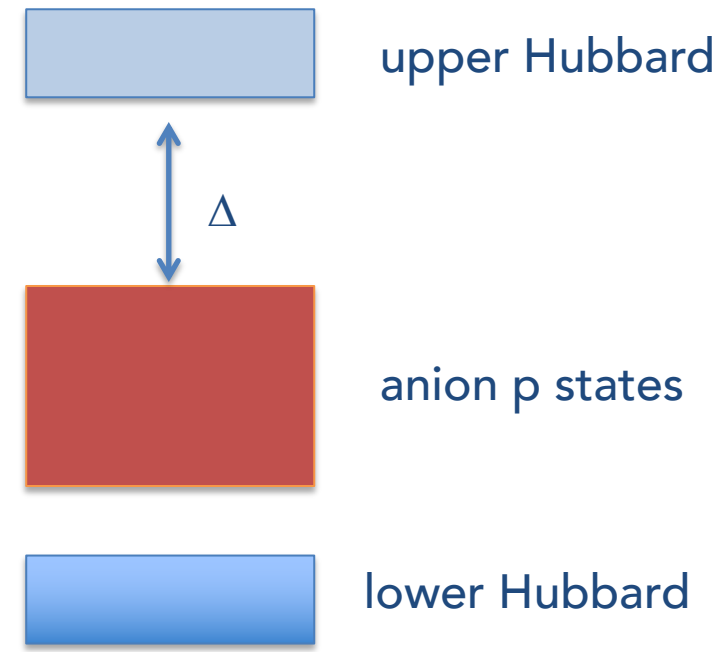
The Zaanen-Sawatzky-Allen phase diagram

A simplified view: In the language of ZSA:

Mott-Hubbard insulators:



Charge-transfer insulators:

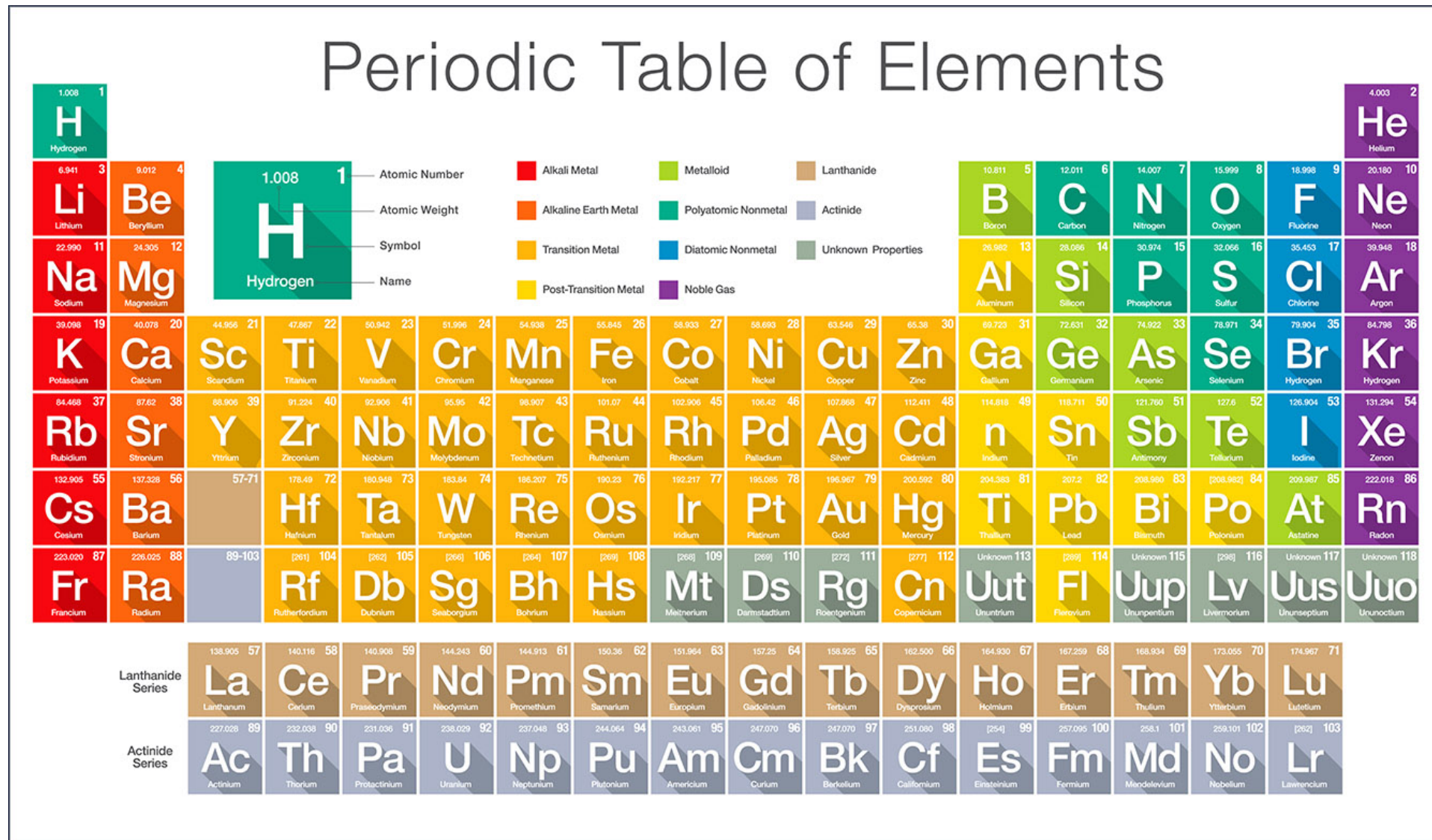


Zaanen, Sawatzky, Allen, *Phys. Rev. Lett.* **55** (1985) 418–421.

Examples of Mott-Hubbard: V_2O_3 , Ti_2O_3 , and most halides. Early transition metals, and lower oxidation states. Interestingly, these display T-dependent M-I transitions.

Examples of Charge-transfer: CuO , $NiCl_2$, NiS , etc. Later transition metals and higher oxidation states. CuO and $NiCl_2$ are always insulating.

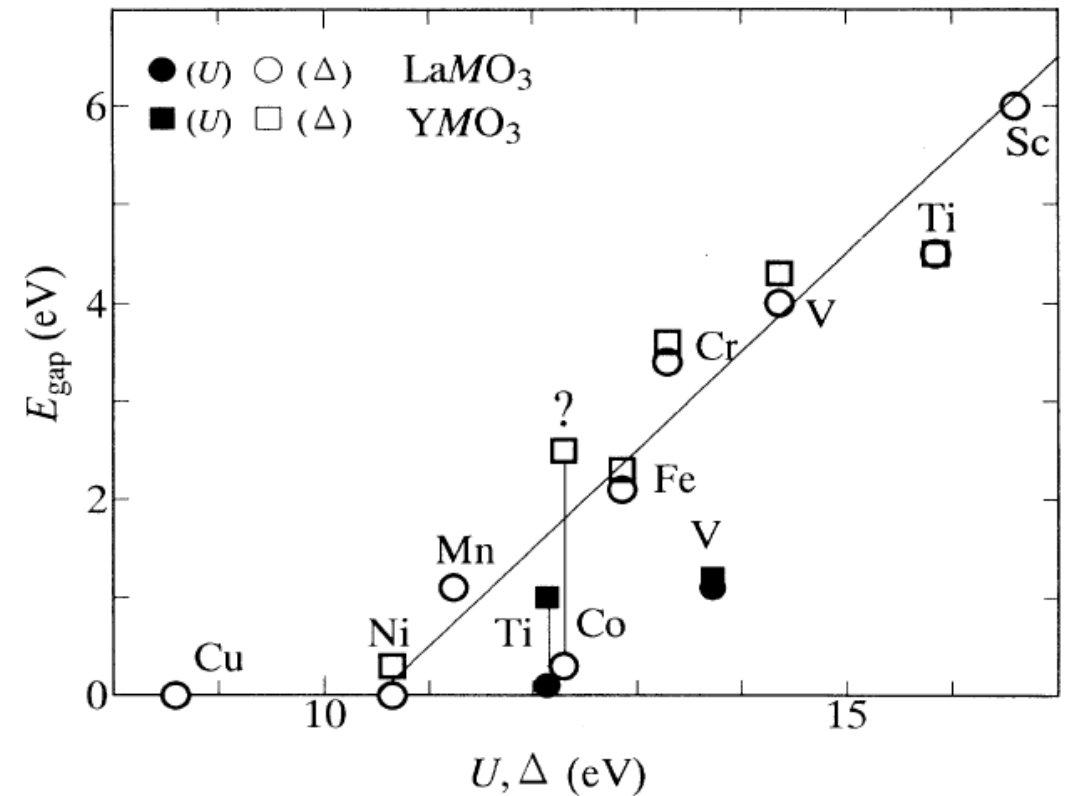
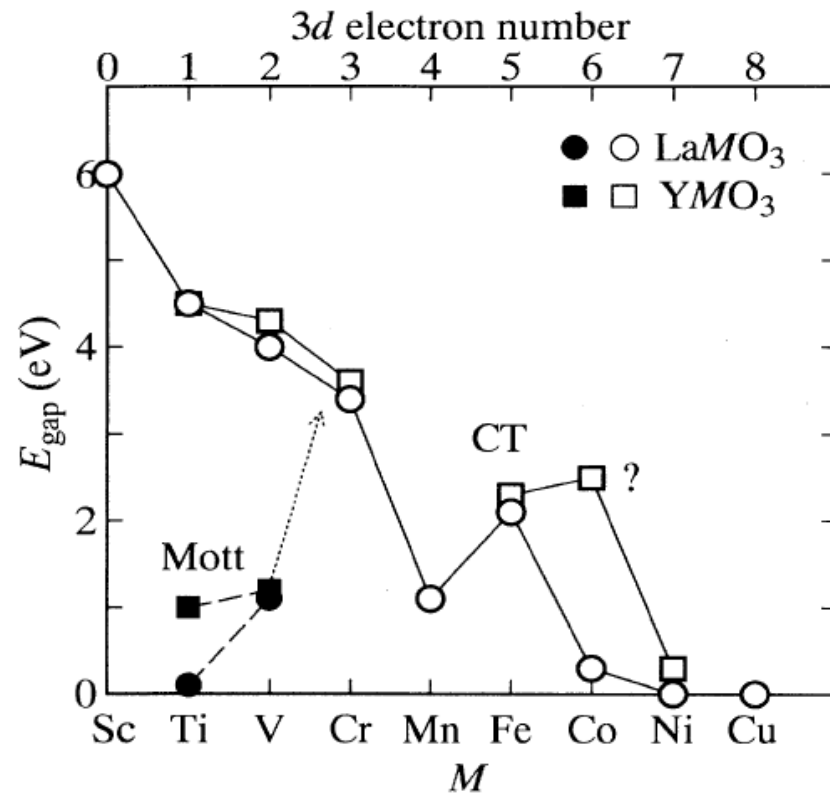
Zaanen, Sawatzky, Allen, *Phys. Rev. Lett.* **55** (1985) 418–421.



Zaanen, Sawatzky, Allen, *Phys. Rev. Lett.* 55 (1985) 418–421.

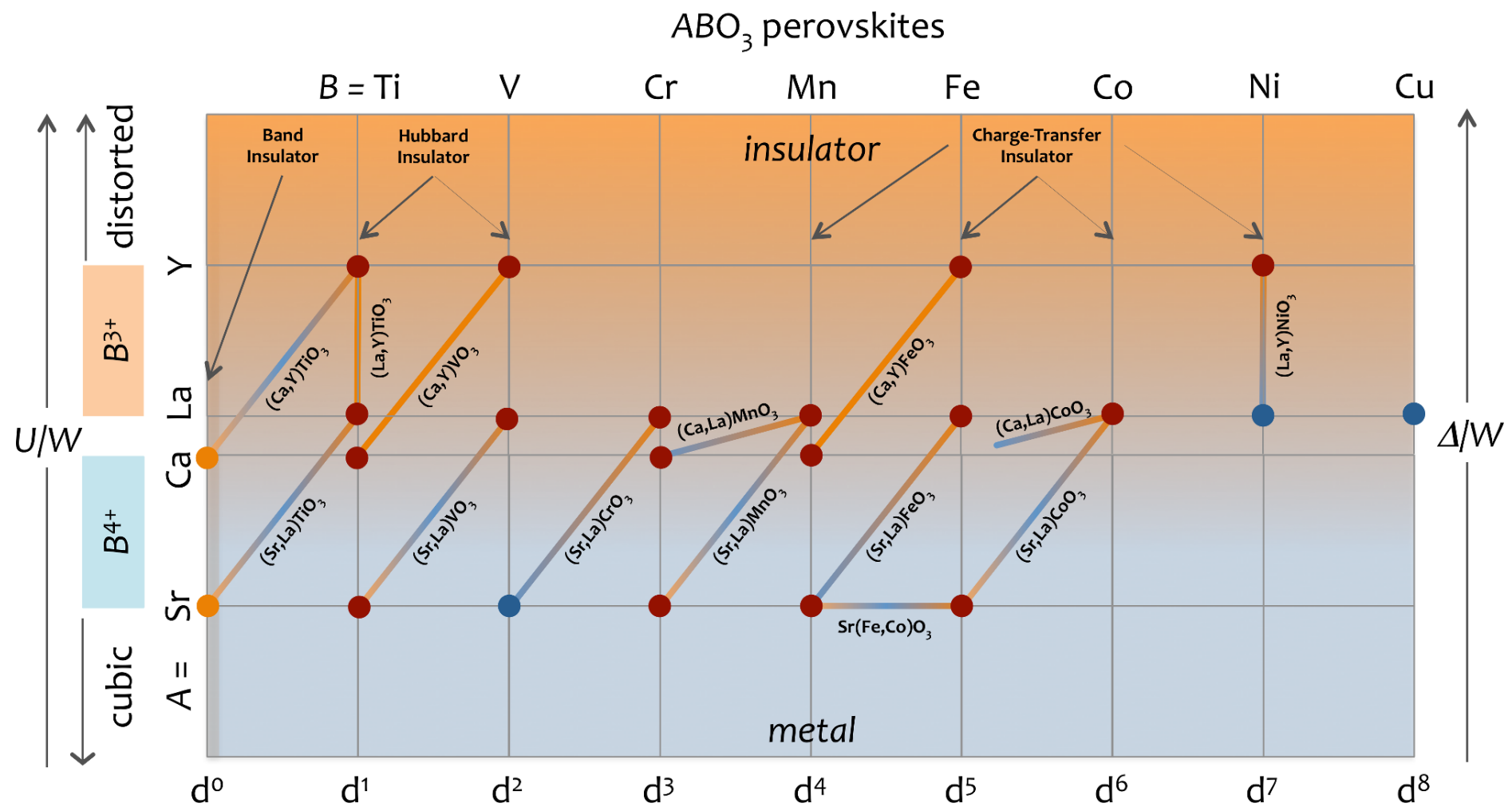
The Zaanen-Sawatzky-Allen phase diagram and perovskites

Optical studies of band gaps: Distinguishing d-d and p-d character:



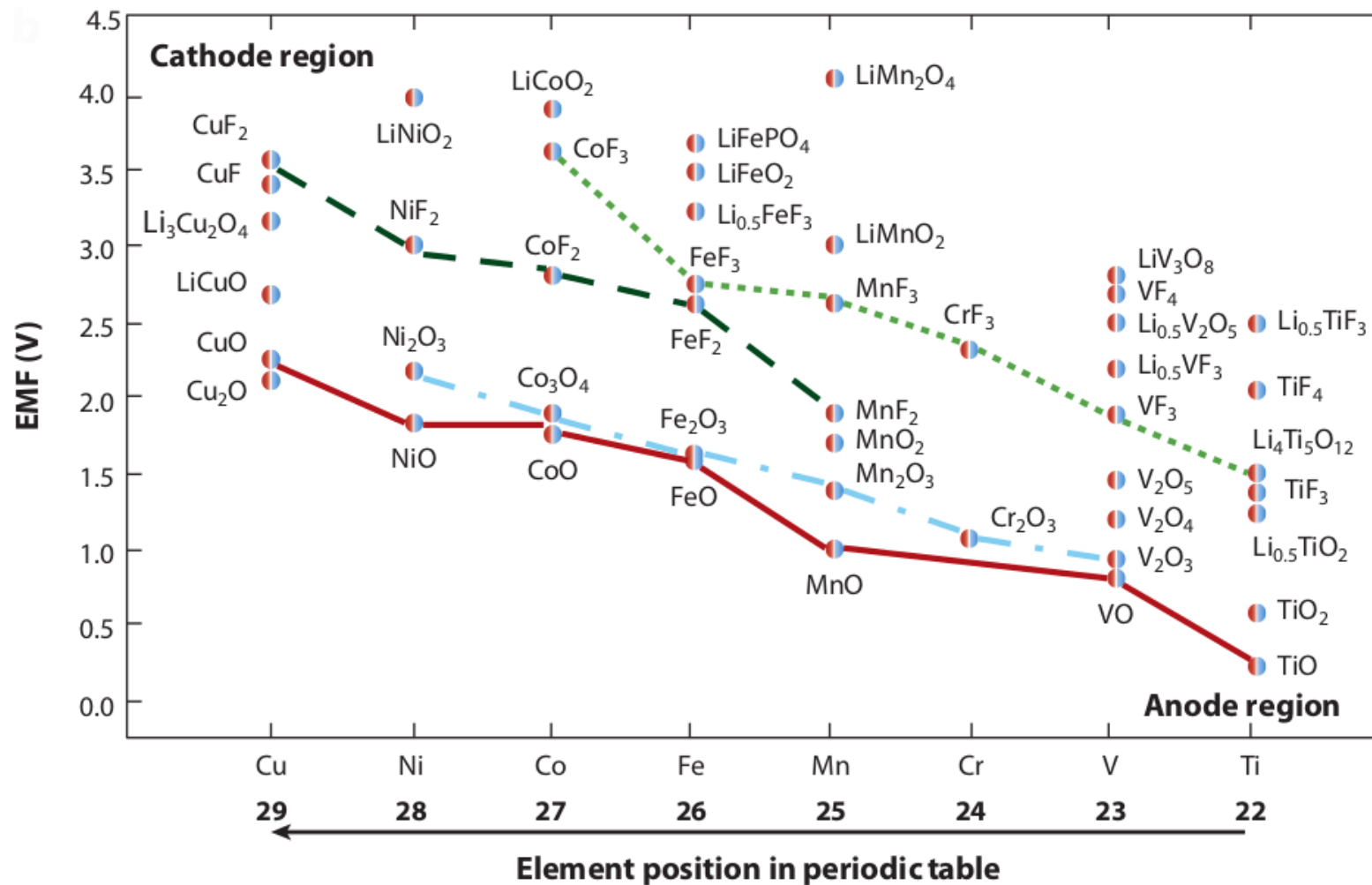
Zaanen, Sawatzky, Allen, *Phys. Rev. Lett.* **55** (1985) 418–421.

The Zaanen-Sawatzky-Allen phase diagram and perovskites



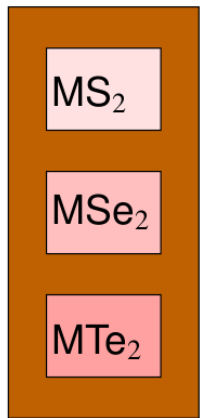
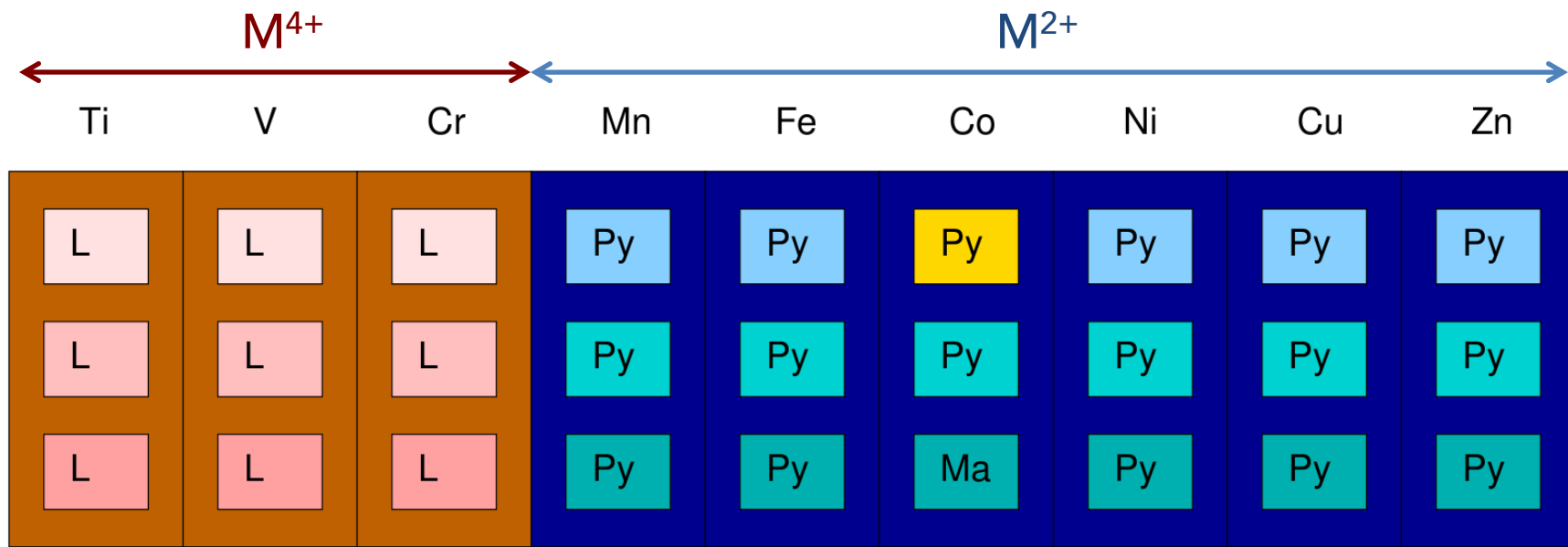
The Zaanen-Sawatzky-Allen phase diagram and batteries

The transition metal d and anion p levels manifest in Li-battery electrochemistry.



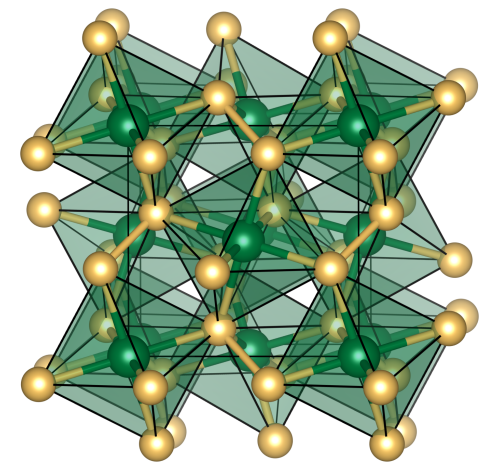
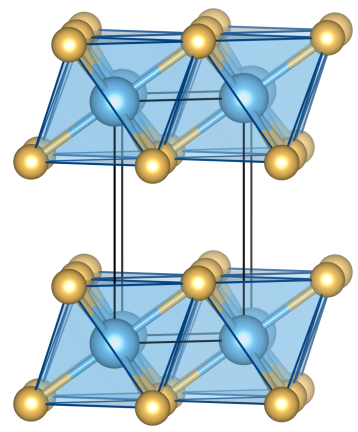
Hayner, Zhao, Kung, *Annu. Rev. Chem. Biomolec. Eng.* 3 (2012) 445–471.

The Zaanen-Sawatzky-Allen phase diagram and redox competition



Py = pyrite
Ma = marcasite
L = layered

Py ferromagnetic



Ideas of Goodenough, Rouxel etc.