Crystal system

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A **crystal system** is a category of space groups, which characterize symmetry of structures in three dimensions with translational symmetry in three directions, having a discrete class of point groups. A major application is in crystallography, to categorize crystals, but by itself the topic is one of 3D Euclidean geometry.

Contents

Overview

There are 7 crystal systems:

- Triclinic, all cases not satisfying the requirements of any other system. There is no necessary symmetry other than translational symmetry, although inversion is possible.
- Monoclinic, requires either 1 twofold axis of rotation or 1 mirror plane.
- Orthorhombic, requires either 3 twofold axes of rotation or 1 twofold axis of rotation and two mirror planes.
- Tetragonal, requires 1 fourfold axis of rotation.
- Rhombohedral, also called trigonal, requires 1 threefold axis of rotation.
- Hexagonal, requires 1 sixfold axis of rotation.
- Isometric or cubic, requires 4 threefold axes of rotation.

There are 2, 13, 59, 68, 25, 27, and 36 space groups per crystal system, respectively, for a total of 230. The following table gives a brief characterization of the various crystal systems:

Crystal system	No. of point groups	No. of bravais lattices	No. of space groups
Triclinic	2	1	2
Monoclinic	3	2	13
Orthorhombic	3	4	59
Tetragonal	7	2	68
Rhombohedral	5	1	25
Hexagonal	7	1	27

Cubic	5	3	36
Total	32	14	230

Within a crystal system there are two ways of categorizing space groups:

- by the linear parts of symmetries, i.e. by crystal class, also called crystallographic point group; each of the 32 crystal classes applies for one of the 7 crystal systems
- by the symmetries in the translation lattice, i.e. by Bravais lattice; each of the 14 Bravais lattices applies for one of the 7 crystal systems.

The 73 symmorphic space groups (see space group) are largely combinations, within each crystal system, of each applicable point group with each applicable Bravais lattice: there are 2, 6, 12, 14, 5, 7, and 15 combinations, respectively, together 61.

Crystallographic point groups

A symmetry group consists of isometric affine transformations; each is given by an orthogonal matrix and a translation vector (which may be the zero vector). Space groups can be grouped by the matrices involved, i.e. ignoring the translation vectors (see also Euclidean group). This corresponds to discrete symmetry groups with a fixed point. There are infinitely many of these point groups in three dimensions. However, only part of these are compatible with translational symmetry: the crystallographic point groups. This is expressed in the crystallographic restriction theorem. (In spite of these names, this is a geometric limitation, not just a physical one.)

The point group of a crystal, among other things, determines the symmetry of the crystal's optical properties. For instance, one knows whether it is birefringent, or whether it shows the Pockels effect, by simply knowing its point group.

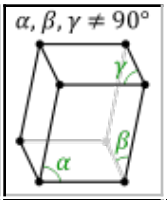
Overview of point groups by crystal system

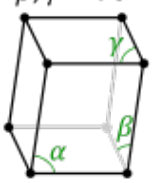
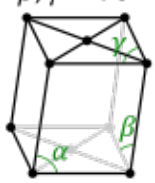
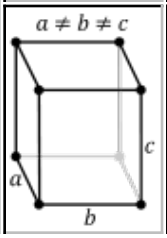
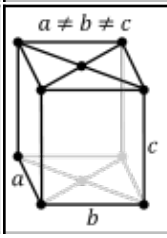
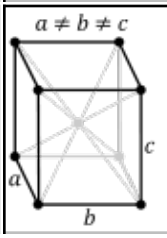
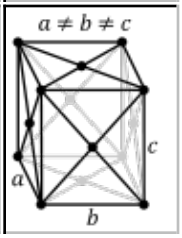
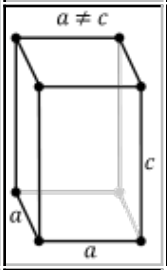
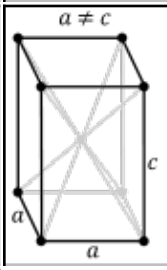
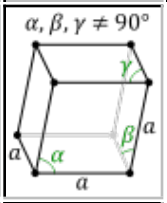
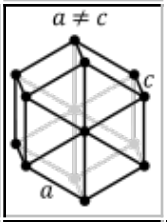
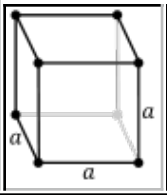
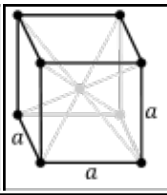
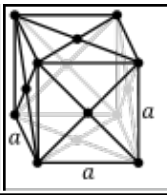
crystal system	point group / crystal class	Schönflies	Hermann-Mauguin	orbifold	Type
triclinic	triclinic-pedial	C_1	1	11	enantiomorphic polar
	triclinic-pinacoidal	C_i	$\bar{1}$	1x	centrosymmetric
monoclinic	monoclinic-sphenoidal	C_2	2	22	enantiomorphic polar
	monoclinic-domatic	C_s	m	1*	polar
	monoclinic-prismatic	C_{2h}	$2/m$	2*	centrosymmetric
orthorhombic	orthorhombic-sphenoidal	D_2	222	222	enantiomorphic
	orthorhombic-pyramidal	C_{2v}	$mm2$	*22	polar
	orthorhombic-bipyramidal	D_{2h}	mmm	*222	centrosymmetric
tetragonal	tetragonal-pyramidal	C_4	4	44	enantiomorphic polar
	tetragonal-disphenoidal	S_4	$\bar{4}$	2x	
	tetragonal-dipyramidal	C_{4h}	$4/m$	4*	centrosymmetric
	tetragonal-trapezoidal	D_4	422	422	enantiomorphic
	ditetragonal-pyramidal	C_{4v}	$4mm$	*44	polar
	tetragonal-scalenoidal	D_{2d}	$\bar{4}2m$ or $\bar{4}m2$	2*2	
	ditetragonal-dipyramidal	D_{4h}	$4/mmm$	*422	centrosymmetric
	trigonal-pyramidal	C_3	3	33	enantiomorphic polar

(trigonal)					
hexagonal	hexagonal-pyramidal	C_6	6	66	enantiomorphic polar
	trigonal-dipyramidal	C_{3h}	$\bar{6}$	3*	
	hexagonal-dipyramidal	C_{6h}	$6/m$	6*	centrosymmetric
	hexagonal-trapezoidal	D_6	622	622	enantiomorphic
	dihexagonal-pyramidal	C_{6v}	$6mm$	*66	polar
	ditrigonal-dipyramidal	D_{3h}	$\bar{6}m2$ or $\bar{6}2m$	*322	
	dihexagonal-dipyramidal	D_{6h}	$6/mmm$	*622	centrosymmetric
cubic	tetartoidal	T	23	332	enantiomorphic
	diploidal	T_h	$m\bar{3}$	3*2	centrosymmetric
	gyroidal	O	432	432	enantiomorphic
	tetrahedral	T_d	$\bar{4}3m$	*332	
	hexoctahedral	O_h	$m\bar{3}m$	*432	centrosymmetric

The crystal structures of biological molecules (such as protein structures) can only occur in the 11 enantiomorphic point groups, as biological molecules are invariably chiral. The protein assemblies themselves may have symmetries other than those given above, because they are not intrinsically restricted by the Crystallographic restriction theorem. For example the Rad52 DNA binding protein has an 11-fold rotational symmetry (in human), however, it must form crystals in one of the 11 enantiomorphic point groups given above.

Classification of lattices

Crystal system	Lattices	
triclinic (parallelepiped)		
	simple	centered

base; here seen from above)	$\alpha \neq 90°$ $\beta, \gamma = 90°$ 	$\alpha \neq 90°$ $\beta, \gamma = 90°$ 		
orthorhombic (cuboid)	simple 	base-centered 	body-centered 	face-centered 
tetragonal (square cuboid)	simple 	body-centered 		
rhombohedral (trigonal) (3-sided trapezohedron)				
hexagonal (centered regular hexagon)				
cubic (isometric; cube)	simple 	body-centered 	face-centered 	

In geometry and crystallography, a **Bravais lattice** is a category of symmetry groups for translational symmetry in three directions, or correspondingly, a category of translation lattices.

Such symmetry groups consist of translations by vectors of the form

$$\mathbf{R} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3,$$

where n_1, n_2, and n_3 are integers and a_1, a_2, and a_3 are three non-coplanar vectors, called *primitive vectors*.

These lattices are classified by space group of the translation lattice itself; there are 14 Bravais lattices in three dimensions; each can apply in one crystal system only. They represent the maximum symmetry a structure with the translational symmetry concerned can have.

All crystalline materials must, by definition fit in one of these arrangements (not including quasicrystals).

For convenience a Bravais lattice is depicted by a unit cell which is a factor 1, 2, 3 or 4 larger than the primitive cell. Depending on the symmetry of a crystal or other pattern, the fundamental domain is again smaller, up to a factor 48.

The Bravais lattices were studied by Moritz Ludwig Frankenheim (1801-1869), in 1842, who found that there were 15 Bravais lattices. This was corrected to 14 by A. Bravais in 1848.

See also

- Crystal structure
- Point group
- Overview of all space groups (in French)
- Overview of all space groups English table

External links

- Overview of the 32 groups (http://newton.ex.ac.uk/research/qsystems/people/goss/symmetry/Solids.html)
- Ditto (http://img.cryst.bbk.ac.uk/sgp/MISC/POINTGRP.HTM) - uses a different notation
- Mineral galleries - Symmetry (http://mineral.galleries.com/minerals/symmetry/symmetry.htm)
- all cubic crystal classes, forms and stereographic projections (interactive java applet) (http://www.ifg.uni-kiel.de/kubische_Formen)

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Categories: Symmetry | Euclidean geometry | Crystallography | Mineralogy

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